Theoretical Approaches for Safety Levels Measurements – Sequential Probability Ratio Test (SPRT)

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The principles of civil aviation safety are considered. At present time, aviation is an ultra-safe system, (i.e. a system that experiences less than one catastrophic safety breakdown every one million production cycles). But, unfortunately, we can never state that aviation activities will be absolutely safe. In current conditions the increase of the civil aviation safety level is one of the principal objectives of the world air transport development.
The paper pays special attention to the civil aviation threats and risks in the context of globalization processes. The investigation deals with problems of global implementation of Annex 19 to the Convention of the International Civil Aviation.
Present research paper discusses new theoretical approaches to searching for methods to assess the "safety space" of civil aviation activity. Special attention is paid to an effective test, proposed by A. Wald.

Keywords: the International Civil Aviation Organization (ICAO), Standards and Recommended Practices (SARPs), safety of aviation, safety management system, financial management, safety space, safety/protection boundary, hypothesis, theory, Sequential Probability Ratio Test (SPRT), accidents and incidents reports, audits, surveys, flight data analysis (FDA), voluntary and involuntary reports and confidential communications.

1. INTRODUCTION

The air transport industry plays a major role in the world economic activity. One of the key elements to maintain the vitality of civil aviation is to ensure safe and sustainable civil aviation operations. The International Civil Aviation Organization (ICAO) sets the Standards and Recommended Practices (SARPs) necessary for aviation safety on a global basis [1 - 2]. The development of global civil aviation safety system unites leading international and regional, intergovernmental and non-governmental organizations, research institutions and universities in order to improve global civil aviation safety level. But contemporary challenges of the world air transport make it necessary to continuously increase the level of safety and security of aviation system. Considering, as it has been mentioned before, the search for new methods to assess the aviation "safety space" seems topical and important for future civil aviation development. This research paper is devoted to the subject and is a logical continuation of the authors' several publications on the issue of development of aviation safety [3 - 12].

2. AVIATION SAFETY MANAGEMENT SYSTEM: THE PROBLEM OF BALANCED ALLOCATION OF RESOURSES

At present time, aviation is an ultra-safe system, (i.e. a system that experiences less than one catastrophic safety breakdown every one million production cycles). Accidents are rare occurrences; consequently their number may vary considerably from one year to the next. As a consequence, on the Figure 1, a 10 year moving average is used by Airbus experts i.e. for any given year, the accident rate is the average of the yearly accident rates over the 10 preceding years. The result of safety aviation activity is a virtually stable absolute number of accidents despite a massive increase in exposure [11,13].

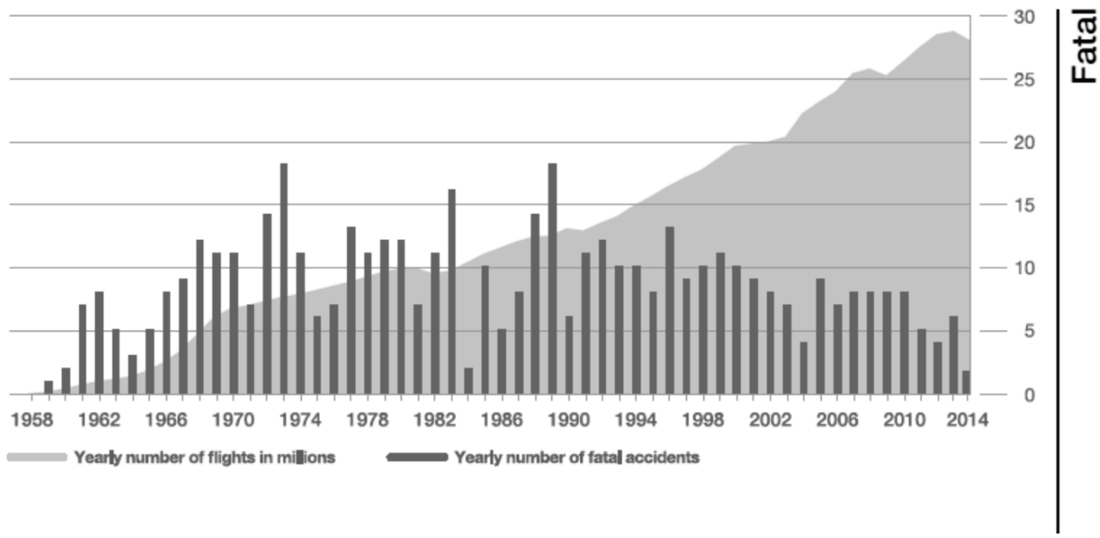

Fig. 1. Yearly Accident Rate Comparing With Number of Flights [13].
(Source: Commercial Aviation Accidents 1958 – 2014. A Statistical Analysis. Airbus S.A.S.).

Nevertheless, unfortunately we can never state that aviation activities will be absolutely safe. Philosophy of liberalization have gradually developed and engulfed more considerable aviation markets during many years. This tendency resulted in the substantial increase in the level of competition among international air routes. But the idea of free competition is not an ideal and can demonstrate stagnation and negative tendencies on different stages of realization in changing conditions. No doubt, on the certain stage their competition is advantageous for customers, passengers or cargo owners. Competition requires from an airline constantly perfecting conditions: high level service and fare policy flexibility. However, the ongoing growth of airlines aircraft operating costs provokes companies to maintain permanent regime of resources economy, which at the same time results in the decline of service quality and in some case in the decline of the acceptable level of safety. On the other hand, permanent exploitation on the verge of profitability or unprofitable routes in general actually put normal existence of airline under a threat [11, 12].

The main trend of the contemporary aviation activity is adoption of a business-like approach to the safety management. The Aviation Safety Management System (SMS) includes business management instruments to the management of safety. The development of business management instruments is aimed at the development of the aviation “safety space”. Within “safety space”, the aviation organization can provide operational activity, with the assurance that it is within a space of maximum resistance to the safety risks or the consequences of hazards. The main boundaries of “safety space” are production and protection (see Figure 2) [2].

From this point of view, it is very important to find the most correct methods of determining these boundaries, in order to prevent the problem of misallocation of resources of aviation organizations. There are two sides to the safety space, or two boundaries: the financial (production) boundary and the safety (protection) boundary [2].

The financial (production) boundary is defined by the financial management of the organization.

We can use the following financial indicators:

- Market trends;
- Changes in prices of commodities;
- External resources [2]

The sources of financial information are:

- Daily collection;
- Analysis of routine financial data [2].

The safety/protection boundary of the safety space should be defined by the safety management of the organization. This boundary prevents incorrect allocation of resources, which may result in a catastrophe.

We can use the following safety indicators:

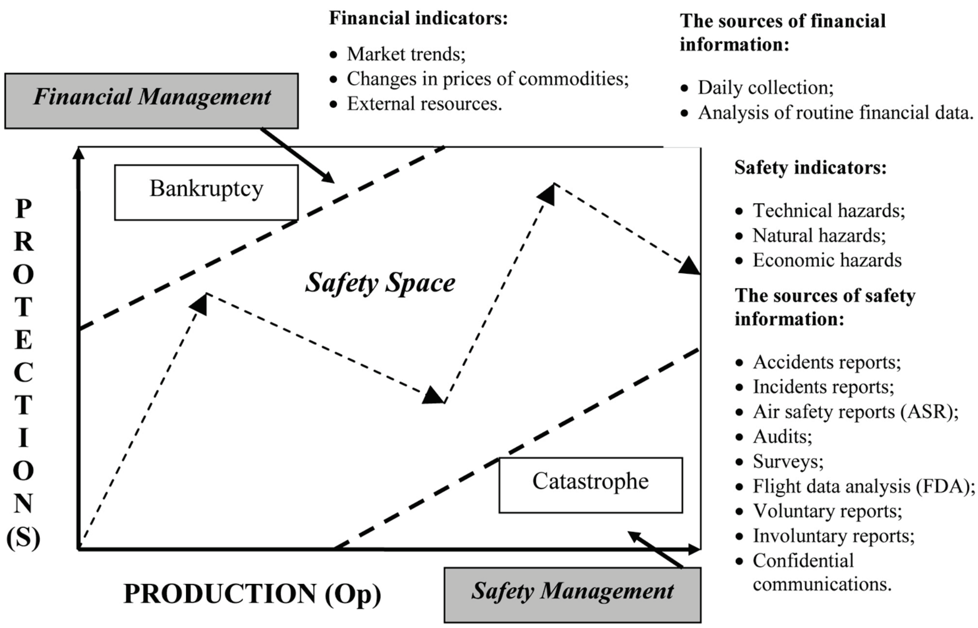

Fig. 2. The safety space.
(Source: Safety Management Manual (SMM): ICAO [2]).

- Technical hazards;
- Natural hazards;
- Economic hazards [2]

The contemporary sources of safety information within Safety Management System are:

- Accidents reports;
- Incidents reports;
- Air safety reports (ASR);
- Audits;
- Surveys;
- Flight data analysis (FDA);
- Voluntary reports;
- Involuntary reports;
- Confidential communications [2].

Having considered the before mentioned information, the search for new methods to assess the aviation "safety space" seems topical and important for future civil aviation safety, efficiency and development.

We present below an effective test, proposed by A. Wald [11,14], which, according to authors, can be very effective for improving the determining of accuracy of the safety area of aviation organization, which is called "safety space".

3. WALD'S SEQUENTIAL PROBABILITY RATIO TEST FOR SOLVING PROBLEM OF BALANCED ALLOCATION OF RESOURSES IN AVIATION

Let X be a random variable with the probability distribution f(x). Let the hypothesis H0 to be tested be the statement that the distribution of X is $f(x,\theta_0)$. Suppose that the alternative hypothesis H1 states that the distribution of X is given by $f(x,\theta_1)$.

If we denote a series of n observations by $X_n = (x_1, x_{2,} ..., x_n)$, then the probability of these n observations is given by

$$P_0 = \prod_{i=1}^{n} f(x_i, \theta_0)$$

if H0 is true
and

$$P_1 = \prod_{i=1}^{n} f(x_i, \theta_1)$$

if H_1 is true.

Current tests of statistical hypothesis assume

1) a doubt about true of the hypothesis
2) possibility of at least two outcomes:
   a) to accept hypothesis,
   b) to reject it

As distinct from current test procedure method of sequential tests of statistical hypothesis is supplemented by the third outcome: «don't know». In such a case the scheme of hypothesis' testing assumes the following form

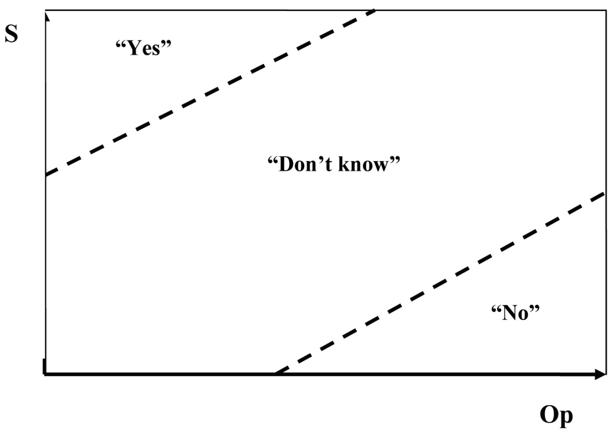

Fig. 3. The scheme of hypothesis' testing assumes [11].

The likelihood ratio may be written in the following way:

$$L = \frac{P_1}{P_0} = \prod_{i=1}^{n} \frac{f(x_i, \theta_1)}{f(x_i, \theta_0)}$$

or in the logarithmic form:

$$\lg L_n = \lg P_1 - \lg P_0 = \sum_{i=1}^{n} \left[\lg f(x_i, \theta_1) - \lg f(x_i, \theta_0) \right]$$

This logarithm of the likelihood ratio $\lg L_n$ is estimated after each i-th experiment and is compared with confidence limits $\lg A$ and $\lg B$ which are determined on the base of the values of the errors of the first type (α) and the second type (β), where

P(reject H_0| H_0 is true) $\leq \alpha$,
P(accept H_0| H_1 is true) $\leq \beta$.

A. Wald has shown that that the lower limit A is given by

$$A = \frac{\beta}{1-\alpha} \leq 1$$

and the upper B is given by

$$B = \frac{1-\beta}{\alpha} \geq 1 .$$

The experiments are continued if there are no sufficient reasons to accept one of two hypotheses, and are used for estimation of the so called likelihood ratio. As it is the ratio of two likelihood functions, one of them corresponds to the hypothesis H_0, the other – to the alternative hypothesis H_1. Then the SPRT procedure is:

At each n-th stage of the sequential analysis

1. If $\lg L_n \leq \lg A$, then the hypothesis H_0 is accepted
2. If $\lg L_n \geq \lg B$, then the hypothesis H_1 is accepted
3. If $\lg A < \lg L_n < \lg B$, take another observation.

For the basic distribution function by special transformations have been obtained the linear equations for lower (S_0) and upper (S_1) limits:

$$S_0(n) = h_0 + kn \text{ and } S_1(n) = h_1 + kn ,$$

where n – is consequently increasing number of experiments and values k, h_0, h_1 are given in the table 1 by analytic expressions.

Furthermore, parameters θ_0 and θ_1 are usually chosen by preliminary considerations.

Then α and β are pre-assigned, k, h_0, h_1 are evaluated for each i-th experiment ($i = 1,2,...,n$ or $i = Z_1, Z_2,..., Z_j,...Z_n$), and

$$S_n = \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} (\sum_{j=1}^{s} x_{ij}),$$

which is compared with the values of s_0 and s_1 evaluated for $n = i$ as shown in Fig.3. One of the hypothesis H_0 or H_1 is accepted as soon as the point with the coordinates (S_i, i) intersects one of the boundary lines.

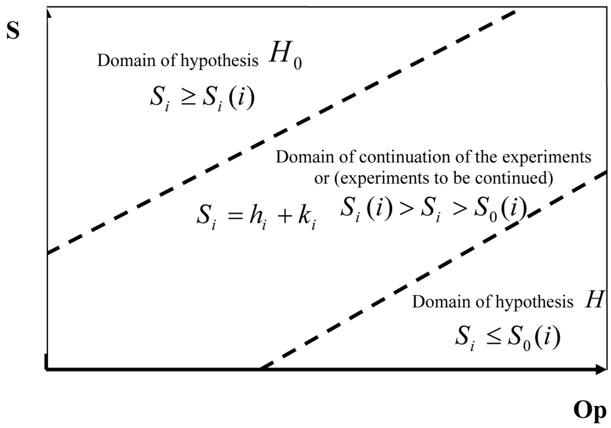

Fig. 4. Explanation of sequential tests of statistical hypothesis [11].

A number of steps ν is a random variable which depends on the «nearness» of the hypotheses (difference: $\theta_1 - \theta_0$), the values α and β and the probability of true or false of the hypotheses.

Practically, by pre-assigned chosen values α and β and correspondence between values of the parameters θ_0 and θ_1 the expectation of the number of steps necessary to fulfil the sequential test procedure $M(\nu, H_0)$ to accept hypothesis H_0 if it is true; $M(\nu, H_1)$ - to accept hypothesis H_1 if it is true and $M(\nu)$ - at worst if $\theta = 0{,}5(\theta_1 + \theta_2)$.

Formulas for pre-assigned estimation of these values are shown in the table 1. The effectiveness of the sequential analysis is estimated by the formula

$$E = 1 - M(\nu) : N$$

Apparently, that if $E > 0$, the sequential procedure is preferable in comparison with the current one if the restrictions allow.

In all examples $S = \sum_{i=1}^{N} x_i = \sum_{i=1}^{n}(\sum_{j=1}^{s} x_{ij})$, when binomial distribution $x_{ij} = \begin{Bmatrix} 0 \text{ an event did not appear} \\ 1 \text{ - an event appeared} \end{Bmatrix}$, when normal distribution and exponential distribution x_{ij} - is indicated value in ij - set when z tests $z \geq 1$ and random values X.

Table 1. Formulae for determination of behaviour area boundary, time-series techniques are used [14].

Type of distribution	Slope ratio k	Absolute term in expression $H_0 : h_0$	Absolute term in expression $H_1 : h_1$
Binomial distribution P_0 и P_1)	$\dfrac{\lg\dfrac{1-P_1}{1-P_0}}{\lg\dfrac{P_1}{P_0} - \lg\dfrac{1-P_1}{1-P_0}}$	$\dfrac{\lg\dfrac{\beta}{1-\alpha}}{\lg\dfrac{P_1}{P_0} - \lg\dfrac{1-P_1}{1-P_0}}$	$\dfrac{\lg\dfrac{\beta}{1-\alpha}}{\lg\dfrac{P_1}{P_0} - \lg\dfrac{1-P_1}{1-P_0}}$
Normal distribution, (Gaussian distribution)	$\dfrac{M_1(X) + M_2(X)}{2}$	$\dfrac{D(X)}{M_1(X) - M_0(X)} \cdot \lg\dfrac{\beta}{1-\alpha}$	$\dfrac{D(X)}{M_1(X) - M_0(X)} \cdot \lg\dfrac{1-\beta}{\alpha}$
Exponential distribution $\lambda = \dfrac{M(X)}{D(X)}$)	$\dfrac{\lg\dfrac{\lambda_1}{\lambda_0}}{\lambda_1 - \lambda_0}$	$-\dfrac{\lg\dfrac{\beta}{1-\alpha}}{\lambda_1 - \lambda_0}$	$-\dfrac{\lg\dfrac{1-\beta}{\alpha}}{\lambda_1 - \lambda_0}$

Table 2. Formula for a priori estimation of middle number of steps (ν) for successive procedure of Wald [14].

Type of distribution	For an acceptance of "zero" - hypothesis, if it is true, $M(\nu, H_1)$	For the acceptance of "alternative" hypothesis, if it is true $M(\nu, H_1)$	In the most unfavourable case $M(\nu)$
Binomial distribution	$\dfrac{(1-\alpha)\lg\dfrac{\beta}{1-\alpha}+\alpha\cdot\lg\dfrac{1-\beta}{\alpha}}{P_0\lg\cdot\dfrac{P_1}{P_0}+(1-P_0)\cdot\lg\dfrac{1-P_1}{1-P_0}}$	$\dfrac{\beta\lg\dfrac{\beta}{1-\alpha}+(1-\beta)\cdot\lg\dfrac{1-\beta}{\alpha}}{P_1\lg\dfrac{P_1}{P_0}+(1-P_1)\cdot\lg\dfrac{1-P_1}{1-P_0}}$	
Normal distribution	$\dfrac{(1-\alpha)\lg\dfrac{\beta}{1-\alpha}+\alpha\lg\dfrac{1-\beta}{\alpha}}{-\left[\dfrac{M_1(X)-M_0(X)}{\sigma(X)}\right]^2}$	$\dfrac{\beta\lg\dfrac{\beta}{1-\alpha}+(1-\beta)\lg\dfrac{1-\beta}{\alpha}}{\left[\dfrac{M_1(X)-M_0(X)}{\sigma(X)}\right]^2}$	$\dfrac{-\lg\dfrac{\beta}{1-\alpha}\cdot\lg\dfrac{1-\beta}{\alpha}}{\left[\dfrac{M_1(X)-M_0(X)}{\sigma(X)}\right]^2}=-\dfrac{h_0h_1}{D(X)}$
Exponential distribution	$\dfrac{(1-\alpha)\lg\dfrac{\beta}{1-\alpha}+\alpha\lg\dfrac{1-\beta}{\alpha}}{\lg\dfrac{\lambda_1}{\lambda_0}-\dfrac{\lambda_1-\lambda_0}{\lambda_1}}$	$\dfrac{\beta\lg\dfrac{\beta}{1-\alpha}+(1-\beta)\lg\dfrac{1-\beta}{\alpha}}{\lg\dfrac{\lambda_1}{\lambda_0}-\dfrac{\lambda_1-\lambda_0}{\lambda_1}}$	$\dfrac{-\lg\dfrac{\beta}{1-\alpha}\cdot\lg\dfrac{1-\beta}{\alpha}}{\left(\lg\dfrac{\lambda_1}{\lambda_0}\right)^2}=-h_0h_1\cdot\dfrac{(\lambda_1+\lambda_0)^2}{4}$

4. CONCLUSION

Application of Sequential Probability Ratio Test is especially relevant for determining the area of the aviation called “safety space”. Due to the different nature of the aviation hazards (technical, natural and economic) and diversity of safety information channels (accidents and incidents reports, audits, surveys, flight data analysis (FDA), voluntary and involuntary reports and confidential communications) the use of Sequential Probability Ratio Test improves the accuracy of “safety space” estimation. It facilitates the search of correct responses to emerging threats and is a prerequisite to further enhancement of the level of civil aviation safety. This will allow to offer more correct responses to aviation safety hazards and is a prerequisite to further reinforcement of the global safety level of the civil aviation.

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