Analysis of the Operation of a Stacker Crane with Simulation Methods

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The paper presents a simulation model of a stacker crane used as a means of internal transport in storage systems. A dynamical model of a stacker crane has been developed in the discrete event driven simulation environment. Results of numerical experiments and time characteristics are presented for various trajectories of a crane.

1. INTRODUCTION

The growing intensity of materials streams, shorter lives of products, ever changing product ranges, decreasing load units and need to reduce warehouse stocks – they all contribute to a steady increase in the number of warehouses and logistics centres at which automated systems have been implemented. Warehouses and logistics centres form a key element of modern delivery chains and can definitely have a material effect on whether a company will succeed or fail.

Internal transport is one of important elements of any warehouse system. Improper management of internal transport results in poor exploitation of operating time of means of transport and increased cost of transport. It also contributes to ineffective use of technological work stands, creation of superfluous stocks at each storage point, as well as the lengthening of order execution time and creation of idle production capacity at an enterprise. Therefore, it is of utmost importance for an internal transport system be appropriately sized and planned, so that specified materials, semi-finished products and ready products may be delivered in required quantities to required destination and at required time.

At the warehouses discussed, stacker cranes have been becoming common transport equipment supporting warehouse processes.

In planning and analysis of transport operations in the logistics and distribution systems, newer still and more versatile tools have been considered. Given the growing expenditure on the implementation of control strategy for materials and information flows in such systems, it appears necessary to perform numerous analyses of the effect of using such strategies on the operation of an entire system, with such analyses supported with simulation methods. Tools designed for quick modelling and simulation and enhanced with interfaces supporting the communication with enterprise management systems, databases, spreadsheets and general-purpose languages enable such analyses as well as testing of practically any control algorithm for materials and information flows.

The paper discusses selected aspects of simulation modelling of internal transport processes, focusing on the analysis of dynamical behaviour of manipulated transport equipment, such as stacker cranes.

2. INTERNAL TRANSPORT SOLUTIONS IN AUTOMATED WAREHOUSE SYSTEMS

As already mentioned above, among internal transport solutions whose use has been becoming more and more common at logistics and distribution centres, there is the implementation of
a system cooperating with a set of stacker cranes, known as automated storage/retrieval systems (AS/RS), which in particular support high-storage racks. The main task of stacker cranes is to transfer load units (pallets, containers etc.) in the rack area of a warehouse. A stacker crane moves in an aisle along a rail mounted in the floor, while the guiding profile is mounted at the top of a rack.

In a typical configuration, a stacker crane may transport at most one pallet. Pallets to be stored are delivered (e.g., by conveyors) to the acceptance station and wait in a buffer (stacking path) until a stacker crane transports them to the specified storage space. Pallets are taken from the buffer on the FIFO (first in, first out) basis: a stacker crane takes this pallet first which has first come into the buffer. At the other end of the warehouse, a pallet, having been located by a stacker crane, is transported to the acceptance/collection station.

In the majority of implementations, a given stacker crane moves along a single aisle; its high horizontal and vertical dynamics, as well as fully automated operation streamline load unit handling within a warehouse.

The literature provides numerous analytic models of stacker crane operation [1, 4, 5, 6, 7]. Below, the analytical model is described based on M. P. Groover’s paper [5]. If \(x\), \(y\) and \(z\) stand for the dimensions of a load unit [in mm] (depth, length and height, respectively), then the width, length and height of the aisle can be expressed as follows:

\[
W = 3(x + a) \tag{1}
\]

\[
L = n_y (y + b) \tag{2}
\]

\[
H = n_z (z + c) \tag{3}
\]

where:

- \(W\) is the aisle width [in mm],
- \(L\) is the aisle length [in mm],
- \(H\) is the aisle height [in mm],
- \(a\), \(b\), \(c\) are respective tolerances for the rack compartment [in mm],
- \(n_y\) is the number of rack compartments along its length,
- \(n_z\) is the number of rack compartments along its height.

The total storage capacity of a single aisle depends on the number of compartments along the rack length and the number of compartment levels, as illustrated in Fig. 1 and can be expressed as:

\[
C_k = 2n_y n_z \tag{4}
\]

The performance of a stacker crane is defined as the number of transactions the crane is able to execute in one hour of operation, with a transaction understood here as storing a load in, or retrieving a load, from a rack. A stacker crane may operate in two modes (respective crane trajectories are shown in Fig. 2):

- **single (simple) cycle**, where a crane exclusively stores a load unit in a rack or exclusively retrieves a load unit from a rack,

- **double (combined) cycle**, where, upon storing a load unit in a rack, a crane retrieves another load from the rack and transports it to the collection zone.

The mode of operation of stacker cranes at high-storage warehouses depends, *inter alia*, on the goods storage method. In modern warehouses, the most common storage methods are:

- **randomized storage** – a pallet may be stored at an arbitrary compartment of a rack,
- **dedicated storage** – goods are assigned to specified compartments,
- **class-based storage** – goods are assigned to racks divided into classes.

![Fig. 1 Visualisation of a warehouse rack with an aisle – side and top view with the stacker crane rails](image-url)
Under the first method, a pallet may be stored at any free storage compartment. This method requires appropriate organisation, monitoring and control of warehouse operation, but the related cost is offset by reduced total storage space.

Class-based storage requires the total storage space to be divided into fields of different turnover ratio. The most common classification criterion is the ABC principle [1, 7, 8].

![Stacker crane travel trajectories](image)

Fig. 2 Stacker crane travel trajectories for a single cycle and double cycle

One of the issues requiring consideration while determining operating parameters of a stacker crane is the analysis of operation cycle durations. The literature describes several methods of determining operation cycle durations and performance for stacker cranes [5, 6, 7]. The method presented below has been recommended by the Material Handling Institute [5]. The method is based on the following assumptions:

- rack compartments are assigned to loads at random,
- vertical and horizontal velocities of a stacker crane are constant,
- a stacker crane is able to move vertically and horizontally simultaneously,
- all compartments of a rack have the same dimensions,
- load acceptance/collection station is situated at the bottom of a rack, at the beginning of an aisle.

The duration of a single cycle [in minutes per cycle] can be expressed as follows:

\[
T_{cs} = 2 \max \left\{ \frac{0.5L}{v_y}, \frac{0.5H}{v_z} \right\} + 2T_{pd}
\]

(5)

where:

- \( T_{cs} \) is the duration of a single (simple) cycle [in minutes per cycle],
- \( L \) is the length of a rack [in mm],
- \( H \) is the height of a rack [in mm],
- \( v_y \) is the horizontal velocity of a stacker crane [in m/s],
- \( v_z \) is the vertical velocity of a stacker crane [in m/s],
- \( T_{pd} \) is the load acceptance or collection time [in seconds].

For a double cycle, the duration [in minutes per cycle] equals:

\[
T_{cd} = 2 \max \left\{ \frac{0.75L}{v_y}, \frac{0.75H}{v_z} \right\} + 4T_{pd}
\]

(6)

where:

- \( T_{cd} \) is the duration of a double (combined) cycle [in minutes per cycle].

The crane performance depends on the proportion of single and double cycles performed. If \( R_{cs} \) and \( R_{cd} \) stand for the number of single cycles and
double cycles, respectively, performed by a crane in an hour, then:

\[ R_{cs} T_{cs} + R_{cd} T_{cd} = 60U \]  

(7)

where:

\( U \) is the indicator of the crane usage per hour; namely, it is the total operation time within an hour.

To solve equation (7), one has to assume the proportion of the \( R_{cs} \) and \( R_{cd} \) indicators (e.g., to assume that the numbers of single and double cycles are equal, i.e., \( R_{cs} = R_{cd} \)).

If \( R_t \) stands for the total number of transactions performed in an hour, then:

\[ R_t = R_{cs} + 2R_{cd} \]  

(8)

3. SIMULATION MODELS OF SELECTED ELEMENTS OF AN INTERNAL TRANSPORT SYSTEM

The high complexity of modern logistics and distribution systems renders infeasible a thorough analysis using exclusively traditional methods based on analytic models. For a comprehensive analysis of materials transport and flow systems, including their dynamics, researchers can use computer simulation methods.

Efficient use of such techniques in support of decision making and analysis of transport and logistics problems requires an efficient simulation tool which would at the same time be easy to use. The DOSIMIS-3® package meets those requirements. It has been widely used in the manufacturing industry, services sector, as well as in education and scientific research.

SIMULATION ENVIRONMENT

DOSIMIS-3® is a module-oriented simulation tool, which has in particular been customised to plan and develop models of internal transport systems and logistics systems [2, 3]. Owing to the in-built library of standard modules representing real elements, the package may prove a powerful tool supporting decision making processes, even in the event of small-scale projects. The simulator function is based on discrete event-controlled processes.

The computations performed by the simulator are based on event service procedures ongoing in the system and time lapse service procedures. The package comprises a simulator, model editor and programme supporting scanning of statistical data. The approach used in the package construction supports the mapping of both static elements (resources) of transport and logistics systems, such as buffers, roller or belt conveyors, shuttle vehicles, work, processing or service stands, switches (turnouts, crossovers), and movable elements.

Table 1 presents the most important modules used in the construction of the model discussed herein.

<table>
<thead>
<tr>
<th>Module Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOURCE – it maps the entry point of material to a system or the entry to a warehouse; it also generates movable objects, such as pallets.</td>
</tr>
<tr>
<td>BUFFER, STACKING PATH – this element represents the route which pallets have to cover and on which objects may be stacked.</td>
</tr>
<tr>
<td>SHUTTLE VEHICLE – this element is to deliver objects (e.g. pallets) from the starting point to the end point, along one of pre-defined routes.</td>
</tr>
<tr>
<td>SINK – the end element of the system, where movable elements flowing through the system come.</td>
</tr>
</tbody>
</table>

Source: Own elaboration

The SHUTTLE VEHICLE is an element deserving special attention; it is the basic element of the model presented herein, supporting mapping of the motion of a stacker crane. Each motion cycle of the shuttle vehicle can be divided into four phases (see Fig. 3). In the first phase (acceleration phase), the vehicle is accelerated to its maximum velocity \( v_1 \). In the second phase, it moves with its maximum velocity and then decelerates to its positioning velocity \( v_2 \). The
positioning having been completed, the vehicle has reached its objective and stops.

From the perspective of analysis of vehicle operation, it is important to include acceleration and deceleration in the analysis, as the values of those parameters have a major effect on the cycle duration for the stacker crane.

![Fig. 3 Phases of the operation of the element SHUTTLE VEHICLE](image)

**SIMULATION ASSUMPTIONS AND DISCUSSION OF A SIMULATION MODEL**

The processes of accepting goods to a warehouse and collection of the goods from the warehouse in response to an appropriate order are treated as two independent processes. It has been assumed that both the orders leading to a stacker crane collecting a pallet from the warehouse and pallets which the crane is to store in the relevant rack in the warehouse appear at random. It has further been assumed that the randomized storage method is used.

The construction of a model starts with introducing appropriate parameters for each element of the model. Table 2 sets forth the list of those parameters, their types, values and descriptions.

**Table 2 Parameters of the simulation model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLACES_X</td>
<td>integer</td>
<td>17</td>
<td>Number of compartments along the length</td>
</tr>
<tr>
<td>PLACES_Y</td>
<td>integer</td>
<td>4</td>
<td>Number of compartments along the height</td>
</tr>
<tr>
<td>CAPACITY</td>
<td>integer</td>
<td>1</td>
<td>Compartment capacity</td>
</tr>
<tr>
<td>LEuro</td>
<td>float</td>
<td>1.2</td>
<td>EuroPallet length [in m]</td>
</tr>
<tr>
<td>HEuro</td>
<td>float</td>
<td>1.8</td>
<td>EuroPallet height (with load) [in m]</td>
</tr>
<tr>
<td>v_std</td>
<td>float</td>
<td>0.3</td>
<td>Standard transport velocity [in m/s]</td>
</tr>
<tr>
<td>v_x</td>
<td>float</td>
<td>2.5</td>
<td>Horizontal velocity [in m/s]</td>
</tr>
<tr>
<td>a_x</td>
<td>float</td>
<td>0.4</td>
<td>Horizontal acceleration [in m/s²]</td>
</tr>
<tr>
<td>pt_x</td>
<td>float</td>
<td>1.5</td>
<td>Horizontal positioning time [in sec]</td>
</tr>
<tr>
<td>v_y</td>
<td>float</td>
<td>0.6</td>
<td>Vertical velocity [in m/s]</td>
</tr>
<tr>
<td>a_y</td>
<td>float</td>
<td>0.9</td>
<td>Vertical acceleration [in m/s²]</td>
</tr>
<tr>
<td>pt_y</td>
<td>float</td>
<td>1.5</td>
<td>Vertical positioning time [in sec]</td>
</tr>
<tr>
<td>PAL_IN</td>
<td>integer</td>
<td>1</td>
<td>Code of pallet being stored</td>
</tr>
</tbody>
</table>

Parameters of the type integer, that is PLACES_X, PLACES_Y, denote the number of compartments along the length and number of compartments along the height, respectively.

For the purposes of the model it has been assumed that a rack has four levels (four compartments along the height) and seventeen compartments along the length, and that one pallet may be stacked in one compartment (a fixed value of CAPACITY). Parameters of the type float, that is v_x, a_x, pt_x, v_y, a_y, denote horizontal and vertical velocities and accelerations, respectively, of a stacker crane.

Fig. 4 presents the chart of the stacker crane model developed in the DOSIMIS-3 environment. When a pallet appears at the entry element (1), a compartment to which it is to be transported (4) is selected at random. If the selected compartment is free, the object (pallet) is transported there; otherwise, the selection is repeated. The source (3) acts as an order generator. If an order appears, a compartment from which a pallet is to be collected is selected at random. If in the selected compartment there is a pallet, it is collected and transported to the exit (2).

The compartments where pallets are to be transported were selected with use of decision tables.

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1 A decision table is a chart which can be treated as a representation of a set of decision rules; these rules are used to map processes involving making decisions.
**SIMULATION RESULTS**

Two modes of stacker crane movement were considered (see Fig. 5).

**DIAGONAL MOVEMENT**

In the diagonal movement mode, the minimum cycle time, is the time of the pallet transport from a buffer to a rack or from a rack to the exit, was 29.40 [s], while the maximum cycle time stood at 58.20 [s]. On average, 324 pallet entries to the warehouse and 317 pallet collections were recorded.

**PERPENDICULAR MOVEMENT**

In the perpendicular movement mode, the minimum cycle time, is the time of the pallet transport from a buffer to a rack or from a rack to the exit, was 33.00 [s], while the maximum cycle time stood at 91.19 [s]. On average, 296 pallet entries to the warehouse and 294 pallet collections were recorded.

Table 3 sets forth statistical data on cycle times in the simulation.

<table>
<thead>
<tr>
<th>Cycle time</th>
<th>Stacker crane movement mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diagonal</td>
</tr>
<tr>
<td>Average</td>
<td>48.00</td>
</tr>
<tr>
<td>Minimum</td>
<td>29.40</td>
</tr>
<tr>
<td>Maximum</td>
<td>58.20</td>
</tr>
</tbody>
</table>

Source: Own elaboration
4. CONCLUSIONS

The role of the transport solutions presented herein in storage and distribution systems has been growing for a time. Given their complexity and effect on the quality of operation of the other system elements, the analysis of these solutions requires appropriate tools. Such tools undoubtedly include discrete event controlled simulation methods. Models of that class, presented in this paper, support mapping and analysis of time relations among the equipment items examined.

Further analysis may be enhanced to include other storage methods and the use of data on actual loads in selected systems. Additionally, the model described can be used as a component of a model of an actual system of logistics or warehouse centre, cooperating with other modules of the system.

BIBLIOGRAPHY


