Simulation Methods Application for LPG Deliveries
Planning and Scheduling to the Network of Stations Under Demand Uncertainty

Marek Karkuła
*AGH University of Science and Technology, Poland*

Jacek Kaleta
*Cracow University of Technology, Poland*

In our paper we considered the problem of demand uncertainty and its influence on planning and scheduling of LPG deliveries. The experience of specialized transportation company in charge of LPG deliveries for the domestic supplier network under VMI approach was analyzed. High variability of distribution parameters and frequent orders modifications were observed while small stations tanks capacities comparing to high daily LPG sales volumes were considered. The combined use of simulation and optimization methods was proposed for the case of LPG distribution to the petrol stations network. The demand uncertainty at customers' stations was considered. Simulation models were assumed to be efficient for dynamic and robust delivery plans of LPG distribution. The results of computational experiments were presented for different values of coefficient of variation.

**Keywords:** VMI, VRP, IRP, DES, variable demand.

1. INTRODUCTION

The significant development of Polish hauliers observed within the last years is linked to the increase of loads volume in domestic and European transport, fleet renewal and service standardization process, initiative of Polish entrepreneurs, foreign logistics operators' investments and last but not least economical advantages of service proposed. As the competition becomes more and more severe, the decision makers are motivated to identify systematically and implement new savings in their business models. Due to stiff competition many companies accept prices that might not allow to generate any profit even if they operate modern fleet of vehicles and provide high level of services. One of the chances to increase profitability while keeping high service level is implementation of planning and scheduling methods that allow transport resources optimization.

The uncertainty of LPG demand at the stations and its impact on the process of inventory routing in a dynamic perspective are the main challenges of this paper. The research was performed basing on the real process observed in case of the highly specialized haulier providing LPG distribution services for one of the domestic petrol stations networks. High variability of delivery parameters and frequent delivery schedule modifications were observed when relatively small storage tanks (comparing to daily sales) were supplied.

The research in supply-chain management provides with an increased recognition that coordination of different functional specialties within a company is mandatory for an integrated plan for the whole chain. Focus on the coordination efforts aimed at the integration of transportation and inventory leads to significant benefits that were well recognized by successful retail businesses such as Wal-Mart, Carrefour, Casino (T. Davis et al.,1999). Implemented by Procter&Gamble, Fruit of the Loom, Shell Chemical, and others, Vendor Managed Inventory (VMI) is an important coordination initiative, where the supplier is authorized to manage inventories of agreed-upon stock-keeping units at retail locations. This idea is known also as Haulier Managed Inventory (HMI) which is a particular case of larger class of supply-chain models called Collaborative Planning, Forecasting and Replenishment (CPFR). In VMI, the vendor assumes responsibility for inventories management.
at retailers’ using advanced tools such as telemetry systems. In VMI, distortion of demand information (known as bullwhip effect) transferred from the downstream supply-chain member (e.g. retailer) to the upstream member (e.g. supplier) is minimized. Thus, stock-out events are less frequent and inventory-carrying costs are reduced. Furthermore, a VMI supplier is obliged to control the downstream resupply decisions (timing and quantities) rather than filling orders as they are placed. Therefore, the approach offers a framework for synchronizing inventory and transportation decisions. In this approach supplier is fully in charge of inventory levels at retailers’ and stock-out elimination is a must. The main challenge for the VMI concept is a complex planning and scheduling process as well as responsibility for stock-out incidents and cost management.

Several models and many solution methods for the VMI concept have been proposed in operational research literature. Most of them are considered as a class of problems called Inventory Routing Problem (IRP). It was the VMI technique in supply-chain management which has driven the research in this area. Many papers are available for the modeling and solution of routing problems and many papers can be found on inventory management models. The aim of IRP is integration of these two areas focusing on the decisions: when to serve the customers, how much product to deliver and how to organize the routes. The case of repeated distribution of single product by homogenous fleet of trailers within definite time horizon could be a good example for the IRP approach implementation. It is assumed that each customer represents a storage tank of known capacity and average daily consumption (demand). The target is to minimize the total distribution costs for a given period so that any stock-out avoidance is a must. Solving IRP problem leads to the inventory replenishment strategy i.e. the detailed product distribution schedule with quantities and precise delivery dates or list of conditions of delivery call out as well as journey plans for each vehicle and for each scheduling period. The main scope of this paper is the combined use of discrete events simulation and optimization methods in order to evaluate inventory routings and delivery plans. It was assumed that simulation models are efficient for dynamic and robust delivery plans of LPG distribution. The results of computational experiments were presented for different values of coefficient of autogas demand variation.

2. LITERATURE REVIEW

Popularity of the VMI strategy in management science as well as supply chain management lead to the development of the class of problems called inventory routing problems (IRP). The algorithms developed by Beltrami and Bodin (1974) seem to be considered as a pioneering approach concerning IRP problems. They were presented in early 1970’s and focused on modeling and simple solution techniques to solve routing problems for the municipal waste collection with time constraints, as well as customers required different days in a week combinations for visits. In the further papers by Fisher et al. (1982) and Bell et al. (1983), a solution for the IRP instance was provided first by applying mixed integer programming. Afterwards, the successful methods to solve large IRP instances were reported by Golden et. el. (1984) and by Dror (1985) who performed investigations of the large liquid propane distribution system to individual and industrial customers. They considered the basic IRP problem components of and they proposed a simulation approach with the vehicle routing problem (VRP) algorithms. Furthermore, they presented the comparison of several different computational schemes and some computation results.

The solution methods of the stochastic version of IRP were the scope of parallel research stream. The paper by M. Dror, G. Laporte and P. Trudeau can be considered as a pioneering work concerning the stochastic IRP. They implemented the rolling horizon approach as a solution method in case of uncertainty of some model parameters. The following IRP research stream was based on the customers’ base split into different delivery groups taking into account their respective demands and other method-specific parameters. Each delivery was aimed to be performed then to all customers of a given group and the routes were determined using the classical VRP or TSP algorithms. This approach was applied, among others, in papers by Anily, S., Federgruen (1990), (1991) and (1993). They proposed several heuristic solutions for IRP problems and proved that the results achieved are asymptotically optimal.

In other paper G. Gallego and D. Simchi-Levi analyzed conditions in which delivery planning to more than one customer at a time could be advantageous. They provided the lower bound on the long run average cost over all inventory-
routing strategies and used it to show how the effectiveness of direct shipping over all inventory-routing strategies depends on the ratio Economic Lot Size of each of the retailers to the vehicle capacity. In further papers J. Bramel and D. Simchi-Levi considerably simplified both the IRP formulation as well as its solution by implementing the location-based heuristics and estimated route calculation for all vehicles.

The beginning of the XXI century provided new algorithms and heuristic methods of IRP problem solving. An important distinguishing in the stock replenishment strategies was proposed - the most common are the two strategies: order-up-to (OU) and maximum level (ML). In the first one each time a retailer is visited, the quantity delivered is such that the maximum inventory level is reached. The second one implies higher flexibility, the only constraint on the shipping quantity is that it must be not greater than the maximum inventory level. OU strategy was applied in the paper of L. Bertazzi, G. Paletta and M. G. Speranza for different formulations of objective function and to study their impact on the final solutions. As a result, the authors were successful to achieve the reduction of number of deliveries, which resulted in a decrease in operating costs.

In some recent papers algorithms providing the exact solution to MVIRP (IRP for multiple vehicles) problems by L. C. Coelho and G. Laporte have been presented. They proposed a model extension for both stock replenishment strategies OU and ML. A further assumption on the symmetry of the transport costs matrix allowed reduction of variables number. They provided solutions for the set of 45 customers in the horizon of 3 days for 3 vehicles.

In one of the last papers Y.-B. Park et al. (2016) proposed a genetic algorithm (GA) for the inventory routing problem with lost sales under a vendor-managed inventory strategy in a two-echelon supply chain comprised of a single manufacturer and multiple retailers. The proposed genetic algorithm was inspired by the solving mechanism of CPLEX for the optimization model. It determined replenishment times and quantities and vehicle routes in a decoupled manner, while maximizing supply chain profits. The proposed genetic algorithm was compared with the optimization model with respect to the effectiveness and efficiency in various test problems. Solutions in a short computational time were found that were very close to those obtained with the optimization model for small problems and solutions that were within 3.2% of those for large problems.

There are a lot of papers in the literature on IRP issues under constant demand. Models that take into account the stochastically variable demand are rarely presented. In previous studies stock-out events are proposed to be solved by relatively simple solutions, for example by generating another distribution route (Cornillier et al., 2012). This is a significant simplification of the planning problem and the results of this approach might be far from the optimal one.

There are few papers in the literature concerning the inventory routing DSS with stochastic parameters (customers' demands in particular) and without the number of simplifying assumptions. For example the paper of W. K. Abduljabbar and M. T. Razma (2012) presents the simulation model and decision support system for the inventory routing problem in case of the upstream supply chain of the oil refinery. In other paper, Cáceres-Cruz et al (2012) presented a discussion concerning the Monte Carlo simulation and some metaheuristics application for the IRP issues. The authors considered a few inventory management strategies for each customer, they presented calculations concerning inventory holding costs depending on the expected demand during the given period, and they estimated the cost savings related to the routes for each of the strategies presented. The use of heuristics and simulation methods can also be found in the paper of Angela and others (2014). The applied models were used to find a rational plan of inventory management policy in retail outlets and inventory routing flows between the depot and the final destinations. Unfortunately not all parameters which are typical to the LPG supply chain have been considered in these papers.

3. AN OUTLINE INFORMATION ABOUT THE LPG MARKET IN POLAND

LPG (Liquefied Petroleum Gas) is a flammable mixture of hydrocarbon gases (mainly propane and butane). It is used as gas but it is stored in tanks in liquid state. LPG is produced during petrol refining process or while starting a new natural petrol exploitation process. It is a very popular energy source e.g. for heating appliances or as a motor fuel (autogas).

Many governments impose less tax on autogas then on petrol or fuel-oil which helps offset the
greater consumption of LPG as alternative option for classical motor fuels. In Poland the average-annual price of autogas represents about half of the price of diesel or EU95 (alternative fuel) since several years. This is a very important incentive for the customers (see POPIHN Annual Report 2015).

Table 1. Yearly average retail prices of fuels and autogas.

<table>
<thead>
<tr>
<th>Year</th>
<th>Diesel</th>
<th>EU95</th>
<th>Autogas</th>
<th>Index*</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>3.67</td>
<td>4.14</td>
<td>1.84</td>
<td>44.40%</td>
</tr>
<tr>
<td>2010</td>
<td>4.28</td>
<td>4.56</td>
<td>2.22</td>
<td>48.70%</td>
</tr>
<tr>
<td>2011</td>
<td>5.06</td>
<td>5.13</td>
<td>2.61</td>
<td>50.90%</td>
</tr>
<tr>
<td>2012</td>
<td>5.69</td>
<td>5.71</td>
<td>2.79</td>
<td>48.90%</td>
</tr>
<tr>
<td>2013</td>
<td>5.50</td>
<td>5.49</td>
<td>2.51</td>
<td>45.70%</td>
</tr>
<tr>
<td>2014</td>
<td>5.22</td>
<td>5.26</td>
<td>2.57</td>
<td>48.90%</td>
</tr>
<tr>
<td>2015</td>
<td>4.48</td>
<td>4.61</td>
<td>1.96</td>
<td>42.52%</td>
</tr>
</tbody>
</table>

Source: own, basing on literature review (POGP Annual Reports)
all numbers in [PLN/liter],
* Index = ratio autogas price to EU95 price

The relatively low autogas prices are the most important factor of LPG sales and increasing number of LPG car installations in Poland since many years. It is estimated that the global number of LPG-propelled motor vehicles in Poland amounted to 15% of all cars while 57% of cars are gasoline powered and 28% of cars are diesel powered.

The most important segment of Poland's global consumption of LPG is autogas, which effectively totaled 1 600 000 tons with its share of nearly 75% and the total number of LPG vehicle refueling stations came to 5 420. It is estimated that in 2015 a total of 70 000 new gas systems were installed in vehicles (net) so the global number of LPG-propelled motor vehicles in Poland amounted to 2 914 000 as at 31st December 2015. Sale of autogas from refueling stations has become an important element of fuel station owners. Large fuel companies which have been restructuring their station networks have virtually completed the process and complemented the stations’ equipment by adding an LPG module. A similar trend has been observed for self-reliant stations as well according to POGP (2015).

There is an important particularity of autogas distribution: usually autogas stations are equipped with relatively small storage vessels face to daily sales volume. Some examples are provided in Table 2.

Table 2. Comparison of LPG storage vessels capacity to the daily sales volumes for some selected autogas stations in Poland.

<table>
<thead>
<tr>
<th>Autogas station</th>
<th>Storage vessel capacity [l]</th>
<th>Daily sales [l]</th>
<th>Ratio capacity / sales</th>
<th>Delivery frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kraków</td>
<td>8400</td>
<td>5500</td>
<td>1,5</td>
<td>Every day</td>
</tr>
<tr>
<td>Cieszyn</td>
<td>5500</td>
<td>3200</td>
<td>1,7</td>
<td>Every day</td>
</tr>
<tr>
<td>Warszawa</td>
<td>5600</td>
<td>3000</td>
<td>1,9</td>
<td>Every day</td>
</tr>
<tr>
<td>Radom</td>
<td>5600</td>
<td>2800</td>
<td>2,0</td>
<td>At least every 2 days</td>
</tr>
<tr>
<td>Bydgoszcz</td>
<td>5600</td>
<td>2800</td>
<td>2,0</td>
<td>At least every 2 days</td>
</tr>
<tr>
<td>Lublin</td>
<td>5200</td>
<td>2400</td>
<td>2,2</td>
<td>At least every 2 days</td>
</tr>
<tr>
<td>Kielce</td>
<td>5600</td>
<td>2500</td>
<td>2,2</td>
<td>At least every 2 days</td>
</tr>
</tbody>
</table>

Source: own

Specific conditions related to LPG distribution are different to those of traditional fuels. Usually the capacity of storage tanks for Diesel or EU95 allows to keep the stock for several days which has an obvious impact on the product distribution.

The analysis of the data presented in Table 2 indicates the need for every day replenishment for the first seven stations, and a frequency of at least once every two days for the other stations. This is due to the limited capacity of storage vessels of those stations and the relatively high level of sales. This is not a common situation for the distribution of other motor fuels (gasoline or diesel), as conventional petrol stations are equipped with underground storage tanks of high capacity which allow for inventory storage for many days.
In the case of LPG stations we usually have to deal with the storage tanks of four basic nominal capacities (2,700 l, 4,850 l and 6,400 l or 9,200 l). Due to limited space at petrol stations it is not possible to enlarge the storage capacity for LPG.

4. DEMAND VARIABILITY PROBLEM

The high variability of demand is one of the most important factors to be considered for the fuels distribution. It depends not only on the seasonality but also on prices variations and some other factors.

Changes in fuel sales at the stations between individual months of 2015 are presented in the diagram in Figure 2. The graph with changes in fuel sales clearly demonstrates monthly sales variations which are the most important for autogas (up to 11% [month/month]). For the year as a whole, the average growth rate of fuel sales at stations was around 2.6 %, whereas diesel sales showed an increase of 2.4 %, petrol a growth of 1.7 %, and autogas an increase of 5.6 %. Based on experience from the practice of fuel distribution it is known that the more detailed analysis of the dynamics of the demand for gas stations can provide with the daily fluctuations of up to more than 50% of the differences between the average daily demand and the real daily sales.

Changes in retail sales at filling stations in Poland in 2015 are presented in Figure 2. The graph with changes in fuel sales clearly demonstrates monthly sales variations which are the most important for autogas (up to 11% [month/month]). For the year as a whole, the average growth rate of fuel sales at stations was around 2.6 %, whereas diesel sales showed an increase of 2.4 %, petrol a growth of 1.7 %, and autogas an increase of 5.6 %. Based on experience from the practice of fuel distribution it is known that the more detailed analysis of the dynamics of the demand for gas stations can provide with the daily fluctuations of up to more than 50% of the differences between the average daily demand and the real daily sales.

It is worth noting that the data aggregated from all the domestic stations networks do not account for individual variations of sales at individual stations. Given the strong dependence of the demand for motor fuels on the retail prices (which are themselves subject to frequent fluctuations resulting, for example, from promotion or response to competition), demand fluctuations observed in practice significantly differ from the average values. These fluctuations were analyzed on the example of the set of 1,200 stations which were supplied by the certain transport company. The coefficient of variation (CV) was calculated for each of them in order to determine the demand for LPG variation at filling stations:

\[ CV = \frac{\sigma}{\mu} \]  

(1)

where \( \sigma \) is standard deviation and \( \mu \) is the average.

The coefficient of variation is the ratio providing information about the dispersion of results, but in relation to how big is the average (median). In our model, each LPG station is characterized by a coefficient of demand variation \( CV_i \) \( (i = 1, \ldots, n) \). We assume demand to be a random variable (stochastic in nature) and no assumption is made about the inventory policy at individual stations.

The results of one year demand variability analysis for the given set of 1,200 LPG stations equipped with telemetry system are presented in Fig. 3.

Fig. 2. Changes in retail sales at filling stations in Poland in 2015. 
Source: POPiHN Annual Report 2015, [month/month as %].
It shows that sales variability of between 40% and 60% (CV values between 0.4 and 0.6) was observed for over a half of all stations. Variability of less than 30% was observed for less than only 2% of the stations. The average value of the coefficient of variation is 0.64 and median 0.54. The values of the first and third quartile are respectively 0.46 and 0.72. For about 10% of the stations the value of the coefficient of variation is higher than 1 (i.e. 100%), which is related to the operational problems, telemetry failures and anomalies in operations.

Future values of demand are mandatory to be determined for the satisfactory distribution service level. Otherwise the efficiency of the delivery process to the stations can decrease, total distribution costs can rise due to stock-out incidents costs or additional deliveries costs and general evaluation of the haulier can be negative.

Daily sales evolution during one year for a given LPG station equipped with telemetry system is presented in Figure 4.
The daily sale values vary from 500 l to 5,700 l. The coefficient of variation for the given set of values is 46% \((CV=0.46)\). LPG demand for the decision maker seems to be non-deterministic and of a high degree of uncertainty.

The above mentioned time series can be considered as a historical demand profile of a given LPG station. Analysis of historical data included in the time series is a common approach for identifying patterns of customer demand and subsequent prediction. Expertise and efficient computer programs are mandatory for complex analysis of the series properties.

5. INVENTORY ROUTING MODEL PROPOSAL FOR AUTOGAS DELIVERIES TO THE NETWORK OF STATIONS UNDER DEMAND UNCERTAINTY

Our discrete programming model is based on the formulation proposed by C. Archetti et al. (2007), further developed by L. Coelho and G. Laporte (2012) for multiple-vehicle version of the problem for the OU policy. The OU policy is assumed to lead to maximum replenishment of stations tank storages what should lead in consequence to decrease the number of deliveries per station.

The problem is defined on an undirected graph \(G=(V, A)\), with \(n\) nodes. The vertex set \(V=\{0,..,n\}\) corresponds to autogas stations and \(V_0\) represents the LPG depot. The arc set \(A=\{(i, j) \mid i, j \in V, i \neq j\}\). Let \(c_{ij}\) denote non-negative cost of travel by arc \((i, j) \in A\), let \(t_{ij}\) denote travel time associated with arc \((i, j) \in A\) and \(d_{ij}\) denote general distance associated with arc \((i, j) \in A\). The cost matrix \(C=(c_{ij})\) is defined on the set \(V\) or on the set \(A\). Its elements \(c_{ij}\) denote routing costs associated with arc \((i, j) \in A\).

Euclidean metric was assumed in the model for the distances between the stations and the LPG depot. It was assumed that autogas deliveries are performed within planning horizon \(t \in T=\{1,..,p\}\) days by identical bulk trucks with a capacity \(Q\) available to provide the service and available for loading at the given LPG depot. They belong to the set of vehicles \(K=\{1,..,K\}\).

The model works with variables:

- \(y_0^t\) - binary variable defined in each time period \(t\) so that \(y_0^t = 1\) if and only if there exists a route to perform in period \(t\);
- \(y_i^t\) - a binary variable defined: \(y_i^t = 1\) if and only if station \(i\) is served in period \(t\);
- \(S\) - subset of stations: \(S \subseteq V \setminus \{0\}\);
- \(h_i\) - unit autogas inventory holding cost on station \(i\) for the given period;
- \(C_i\) - storage tank capacity of station \(i\);
- \(r_i\) - autogas inventory level at the LPG depot in period \(t\);
- \(I_0^i\) - autogas inventory level at the LPG depot at the beginning of the planning horizon;
- \(I_i^t\) - autogas inventory level at station \(i\) at the beginning of the planning horizon;
- \(I_0^t\) - autogas inventory level at the LPG depot at the end of period \(t\);
- \(I_i^t\) - autogas inventory level at station \(i\) at the end of period \(t\);
- \(d_i\) - autogas demand at station \(i\) in period \(t\);
- \(q_i\) - quantity of autogas delivered to the station \(i\) in time period \(t\);

The objective of the problem is to minimize the total inventory-distribution cost while meeting the demand of each customer. The replenishment plan is subject to the following constraints:

- the inventory level at each customer can never exceed its maximum capacity;
- inventory levels are not allowed to be negative;
- the supplier’s vehicles can perform at most one route per time period, each starting and ending at the supplier;
- vehicle capacities cannot be exceeded.

The solution to the problem determines which customers to serve in each time period, which of the supplier’s vehicles to use, how much to deliver to each visited customer, as well as the delivery routes. The problem consists of minimizing the objective function (2):

\[
\sum_{i=0}^{n} \sum_{t=1}^{p} h_i I_i^t + \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} \sum_{t=1}^{p} c_{ij} x_{ij}^t \tag{2}
\]

subject to constraints:
\[ I_0^t = I_0^{t-1} + r^t - \sum_{k=1}^{K} \sum_{i=0}^{n} q_i^{kt} \]  \( \forall t = 1, \ldots, p \) \tag{3}

\[ I_0^t \geq 0 \quad \forall t = 1, \ldots, p \] \tag{4}

\[ I_i^t = I_i^{t-1} + \sum_{k=1}^{K} q_i^{kt} - d_i^t \] \( \forall i = 1, \ldots, n, \forall t = 1, \ldots, p \) \tag{5}

\[ I_i^t \geq 0 \quad \forall i = 1, \ldots, n, \forall t = 1, \ldots, p \] \tag{6}

\[ I_i^t \leq C_i \quad \forall i = 1, \ldots, n, \forall t = 1, \ldots, p \] \tag{7}

\[ \sum_{k=1}^{K} q_i^{kt} \leq C_i - I_i^{t-1} \] \( \forall i = 1, \ldots, n, \forall t = 1, \ldots, p \) \tag{8}

\[ q_i^{kt} \geq C_i y_i^{kt} - I_i^{t-1} \] \( \forall i = 1, \ldots, n, \forall k = 1, \ldots, K, \forall t = 1, \ldots, p \) \tag{9}

\[ q_i^{kt} \leq C_i y_i^{kt} \] \( \forall i = 1, \ldots, n, \forall k = 1, \ldots, K, \forall t = 1, \ldots, p \) \tag{10}

\[ \sum_{i=1}^{n} q_i^{kt} \leq Q_k y_0^{kt} \] \( \forall k = 1, \ldots, K, \forall t = 1, \ldots, p \) \tag{11}

\[ \sum_{j=1}^{n} x_{ij}^{kt} + \sum_{j=1}^{n} x_{ji}^{kt} = 2y_i^{kt} \] \( \forall i = 1, \ldots, n, \forall k = 1, \ldots, K, \forall t = 1, \ldots, p \) \tag{12}

\[ \sum_{i \in S} \sum_{j \in S} x_{ij}^{kt} \leq \sum_{i \in S} y_i^{kt} - y_m^{kt} \] \( S \subseteq V', m \in S, \forall k = 1, \ldots, K, \forall t = 1, \ldots, p \) \tag{13}

\[ q_i^{kt} \geq 0 \quad \forall i = 1, \ldots, n, \forall k = 1, \ldots, K, \forall t = 1, \ldots, p \] \tag{14}

\[ x_{ij}^{kt} \in \{0,1,2\} \] \( \forall i = 1, \ldots, n, \forall k = 1, \ldots, K, \forall t = 1, \ldots, p \) \tag{15}

\[ x_{ij}^{kt} \in \{0,1\} \] \( \forall i = 1, \ldots, n, \forall j = 1, \ldots, n, \forall k = 1, \ldots, K, \forall t = 1, \ldots, p \) \tag{16}

\[ y_i^{kt} \in \{0,1\} \] \( \forall i = 1, \ldots, n, \forall t = 1, \ldots, p \) \tag{17}

Constraints (3) define the inventory at the LPG depot while constraints (4) prevent LPG stock-outs at the depot. Constraints (5) and (6) are similar and apply to the autogas stations. Constraints (7) impose maximal inventory level at the autogas stations. Constraints (8)–(10) link the quantities of LPG delivered to the routing variables - in particular they only allow the bulk truck to deliver LPG to the station if the station is visited by this bulk truck and enforce the OU policy. Constraints (11) ensure that vehicle capacities are respected, constraints (12) and (13) are degree constraints and sub-tour elimination constraints. Constraints (14)–(17) enforce integrality and non-negativity conditions on the bulk trucks.

6. INVENTORY ROUTING MODEL PROPOSAL FOR AUTOGAS DELIVERIES TO THE NETWORK OF STATIONS UNDER DEMAND UNCERTAINTY

In this section the analysis of the combined use of mathematical models (MIP) and discrete event simulation models was conducted. This combination of optimizing methods with
simulation models can contribute to the formation of more accurate and more robust plans. Developed simulation models will allow to study the impact of demand variability on distribution costs.

The proposed approach will also allow:
− simulating generated routes under **realistic** (stochastic) demand conditions,
− analysis of impact of **demand variability** on distribution costs,
− simulation can effectively handle the **complexity of systems**, 
− useful in case of **uncertainties** (e.g. travel time) in the distribution environment,
− simulation scenarios – data for possible re-optimization.

In order to establish information exchange between models, the possibility exists of controlling simulation runs via COM (Component Object Model) interface supported by both the optimization tool and by the simulator. COM interface developed by Microsoft enables software components to communicate in the MS Windows family of operating systems.

Discrete event simulation (DES), that enable the analysis of the dynamic behaviour of the system and the detailed study of the parameters associated with the different states of the system entities. In case of applying a discrete event simulation methods it is possible to carry out thorough researches and experiments on the models taking into account the detailed elementary operations and information on transport processes.

Discrete event simulation methods are used in modern simulation packages to which the DOSIMIS-3 packet belongs. It is a graphical, interactive package used for modelling, among others, logistics and transport systems. By using standard modules as sources, sinks, processing stations, vehicles, buffers, the user in a relatively simple manner can represent transport system elements. Available modules (representing the emergency conditions and down times) allow for the analysis of blockages, interferences or disruptions in objects flow, analysis of their causes and locations. Also the processes of loading and unloading the load to and from transport vehicles, as well as the processes of reloading and repackaging may subject to modelling in the DOSIMIS-3 environment.

The input data used in the models of distribution operations and processes include the distances between the vertices of the network, data on transported goods (number and place of source and sink, time data), the vehicle data (type, capacity, speed, etc.) and customer demand data (randomly generated demand with different variability).

In the simulation model built in the used simulator, the equivalent of graph representation is the representation of the transport network with the use of predefined elements that reproduce the functionality and logics of the actual internal transport infrastructure elements. The relationship of the mathematical model elements in the form of a graph and the simulation model built in an integrated modelling environment are shown in Fig. 5.

![Fig. 5. Illustration of relations of components in transport system models: graph model and simulation model built in the simulator environment. Source: own.](image_url)
Simulation tool and description of the DES model

As the basic modelling environment the DOSIMIS-3 simulator has been chosen. This package is a module-oriented simulation tool, adapted to, inter alia, designing and developing models of logistics and transport systems. Owing to its module-oriented approach to a modelling problem, DOSIMIS-3 quickly delivers reliable results and may be an excellent tool supporting decision making, even in minor projects. The DOSIMIS-3 package is an interactive, graphic discrete event simulator and is able to simulate various systems, including complex logistics systems and processes.

The package is a discrete, event-oriented simulation program – the simulator inspects those points in time only at which events take place within the simulation model. DOSIMIS-3 has several dozen of predefined components which cover several modelling levels: material flow level, organisational level and control level. In order to understand and analyse logistics systems and processes, one should be familiarised with essential simulation components. The significant components of discrete-event simulation models are:

- **modules (entities):**
  - they represent behaviour of static elements or resources of a system,
  - module has a specified process logic,
  - modules may represent, e.g., buffers, workstations, loading or unloading stations, assembly and disassembly places and many others.

- **(movable) objects or transfer entities:**
  - they are used to describe movable parts, products, vehicles, people or a piece of information.

- **junctions:**
  - they transmit information from one module to the adjacent one.

For the modelling of logistics systems and processes, DOSIMIS-3 uses components which can perform the following functionalities:

- **at the material flow level** – accumulate paths, distribution cars, workstations, buffer capacities, assembly and disassembly stations,
- **at the control level** – decision tables, bottleneck controls, signals, monitoring components,
- **at the organisational level** – disturbances, maintenances, set-ups, breaks and labour organisation.

The required behavior of the modeled system may be enhanced by applying attributes to describe detailed and actual properties of systems' components. Attributes are values, which can be assigned to the objects or the modules via decision tables. These are referred to directly during the evaluation of the parameters without a further decision table being necessary. In our case, important attributes of the simulation model are:

- **customer’s attributes:**

![Fig. 6. Simulation model developed in DOSIMIS-3 tool. Source: own.](image-url)
- (stochastic) demands,
- (stochastic) service times,
- delays,

- vehicle’s attributes:
  - capacity,
  - actual stock,
  - commercial speed.

Enhancing simulation models – application of decision tables

The mapping of the transport systems often requires special strategies. For example the management of the vehicles would have to be organized globally and the disposition of the transport orders should be realized in some special way. In this case, the functionality of the built transport system models can be extended by using the decision tables (DT). Decision tables describe the logics of the system operation or the process realization depending on the occurring conditions and respective actions in the system. The decision tables define the conditions and describe under which circumstances they will be executed. The table also contains instructions concerning the behaviour of particular element of the model, when the condition is met or not. These mechanisms are very useful for testing complex algorithms controlling the flow of mobile objects (loads, load units, vehicles, persons, etc.) and information flow and they are usually used when it is not possible to realize the assumed objects flow or model behaviour using the standard parameterization and strategy. The decision table notation is borrowed from symbolic logic, and it is developed specifically for solving data processing problems. They are designed to make simple, clear, unambiguous statements. They allow to reach a precise and solid way to represent sophisticated policies, control algorithms and business logic. DTs, similar to if-then-else and switch-case statements associate conditions with actions to perform. One of the main advantages of using decision tables as a specification method is that the sophisticated flow logic can be expressed in a compact form in a table by combining rules. The following elements are included in the formal description of the classic decision table:

- condition set $CS = \{CS_1, CS_2, ..., CS_n\}$ – it determines the finite set of possible conditions for the specified decision situation (e.g. condition $CS_1$ – vehicle arrives at the gas station, condition $CS_2$ – the vehicle is carrying a $X$ type load);

- condition domain $CD = \{Y, N\}$ – a set of possible values that can be assigned to the conditions; usually it constitutes the two-element subset of logic values $Y$ (Yes) and $N$ (No); the number of conditions in decision tables determines unequivocally the conditions space – which is the subset of all combinations: $CSP = \{Y, N\}^k$ – as an example for the decision table, represented by four conditions, the number of combinations of equals $2^4 = 2^4 = 16$;

- action set $A = \{A_1, A_2, ..., A_m\}$ – it determines the finite set of possible actions that will be taken in case of meeting or not meeting the decision rules;

- action domain $AD = \{\{Y, N\}\}$ – similarly as for conditions domain – it determines the set of possible values that will be assigned to the action.

Fig. 7. Attributes assigned to different components of developed mode.
Source: own.
Using decision tables for solving particular problems, it is possible to integrate both qualitative and quantitative models that perform some calculations (e.g. to determine the value of a condition variable or to execute an action). Generation of a demand at customer taking into account the variability can be implemented with the aid of decision table in the following way:

- **condition**: act\_module.entering = 1 – condition refers to the event of arrival of the vehicle to the customer’s station, if value of condition is true the following action is executed;
- **action**: act\_module.floatatt.act\_demand:= abs(normally\_dist(act\_module.float\_att.demand,act\_module.float\_att.demand,act\_module.float\_att.demand)*floatpar(CV)) – an action for generating real demand upon arrival of the vehicle

An important step in constructing decision tables used in the simulation model is the procedure of verification and validation of implemented algorithms. Within the frame of this procedure, first of all the consistency of decision table is checked – the decision table is inconsistent (or it contain contradiction), if there is a pair of overlapping rules and the corresponding actions are different. In addition, the completeness, exclusivity and inclusiveness of rules contained in decision table are examined.

7. METHODOLOGY OF COMBINING SIMULATION AND OPTIMIZATION MODELS FOR THE ROUTES AND DELIVERY DATES

In our approach we consider combined use of simulation and optimization models for robust solving of inventory routing problem of LPG deliveries to petrol stations within one region in Poland (one depot, multiple vehicles, several dozen customers). Starting from the optimisation model in order to determine an optimal or suboptimal MIIRP solution of the problem, our approach proceeds with application of discrete event simulation tool (DES), which is able to provide information about whole system behaviour and its reactions to LPG demand variations at petrol stations (see Fig. 8). The solution generated by optimisation model is used as input for simulation model to verify the feasibility and robustness of computed solution through the generation of different scenarios which consider different levels of demand variability typical to real life systems.

The results of the simulation experiments, allowing an evaluation of the system performance, can support the detection of the current solution weaknesses and limitations of the initial problem MIIRP solution. The feedback loop is then realised going back to the optimisation phase with the new information generated by the simulation model. This information is used to improve the initial optimal or suboptimal solution. This approach allows to evaluate the relevant system performance in case of different levels of demand variations and indicate weather re-planning during the day is necessary and possible.

![Overview schema of simulation-optimization integrated approach.](source: own)

In order to describe the demand variability at the stations we apply coefficient of variation (CV) which is defined as the ratio of the standard deviation σ to the mean μ:

\[
CV = \frac{\sigma}{\mu}
\]  

(20)

It shows the extent of variability in relation to the mean of the population. In our model each node shall represent each petrol station with specific coefficient of variation \(CV_i\) \((i = 1, ..., n)\). Final demands at the petrol stations are assumed to be random variables (stochastic in nature) and no
assumption is made about the inventory policy at an individual station. The inventory position of each station shall be analyzed and joint inventory and routing decisions shall be made to avoid any stock-outs and minimize the total expected cost of transportation in each scenario (which is considered as linear function of total travelling distances). The inventory cost of LPG at stations shall be as the cost of LPG is rather low (comparing e.g. to other liquid fuels) and usually stations' vessels capacities are relatively low.

8. NUMERICAL EXPERIMENTS

In this section, some results of computational experiments are presented to illustrate the combined use of proposed optimization and simulation models. For the computational experiment real data LPG distribution within one region in Poland for a given gas supplier has been taken into consideration. The region consists of one depot and 51 gas stations 24/7 open. Traditionally the customer uses from 4 to 5 trucks to organize deliveries to the stations during 7 days per week and considers cost efficiency (number of kilometres per 1000 L delivered) as key point.

The following assumptions have been considered:

- number of customers: 51;
- number of depots: 1;
- planning horizon: 3 days;
- demand distributions: average demand is generated as an integer random number following a discrete triangular distribution with lower limit 0% and upper limit 85% due to some LPG technical properties;
- product availability at the depot: always;
- maximum inventory level: 85%;
- starting inventory level: randomly generated;
- vehicle capacity: 36,000 l.

In our approach the inventory holding costs are negligible as LPG unitary cost is much lower than the cost of other fuels and capacities of stations' tanks are generally small comparing to other fuels tanks. The number of bulk trucks dedicated for the distribution within the given region (e.g. 5 tucks) has been calculated as the ratio of the average daily sales of all the station of the region divided by single bulk truck capacity.

Our aim is to minimize number of kilometers per 1000 L of LPG delivered and avoid any stock-out at any station. Simultaneously we control the aggregated volume of LPG in the whole distribution network of station: we compare the total daily sales of all stations with the volume of gas delivered to the stations within one day. Our aim is to avoid situation when the aggregated volume of LPG in whole distribution network would drop below a certain critical level which shall oblige us to consider much higher number of trucks and drivers to prevent our supply chain from general inefficiency. We assume maximum bulk trucks utilization (performance) and routing optimization.

The key performance indicators (KPIs) of our model are:

- bulk trucks utilization level – $\lambda$ [%];
- number of stock-out events occurred during the whole distribution period – $\epsilon$;
- number of kilometers per tons of LPG delivered - $\alpha$ [km/t];
- stations filling level - $\Omega$ [%];
- average distance travelled per bulk truck – $\mu$[km];
- average number of stations per journey – $\gamma$;
- average drop size per station (EOQ model) – $\beta$ [L].

In Table 3 we provide results for combined use of simulation and IRP optimization models for planning horizon of three days. For different coefficient of variation levels we present results for the above mentioned KPIs.

Some conclusions can be drawn from Table 3. All experiments have been performed 10 times for each value of CV. The values in Table 3 are average per trip. The bulk truck utilization level was very high - at least 96% and in most of the cases 100%. The service level was also high - two stock-out events occurred and only for higher values of CV (0.4 and 0.5). It looks reasonable as probability that demand can be higher than stock level increase for higher values of CV. For higher CV values: distribution routes are longer, average number of deliveries per trip increase and average drop size decrease. The average number of kilometers per ton of LPG delivered varied from 17 to 24 km/t (higher values were observed for higher CV levels).
Table 3. Results of optimization/simulation experiments for the 3-days planning horizon.

<table>
<thead>
<tr>
<th></th>
<th>Coefficient of Variation (CV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda$ [%]</td>
<td>100</td>
</tr>
<tr>
<td>$\epsilon$ [%]</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$ [km/\text{t}]</td>
<td>19.23</td>
</tr>
<tr>
<td>$\Omega$ [%]</td>
<td>40.45</td>
</tr>
<tr>
<td>$\mu$ [km]</td>
<td>384.29</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>5</td>
</tr>
<tr>
<td>$\beta$ [L]</td>
<td>7200</td>
</tr>
</tbody>
</table>

Source: own

9. CONCLUSIONS AND FURTHER RESEARCH DIRECTIONS

The problem of LPG distribution optimization is important in the economy due to the constantly growing volume of LPG sold and lack of decision-support tools for the hauliers. Although there are many papers on delivery planning and scheduling in the literature, most of the available algorithms are not applicable for the LPG distribution. Traditional challenges of inventory management are usually different than in the case of LPG.

The specific decision-support model was proposed as the specificity distribution network of autogas stations requires a special approach. The simulation experiments basing on delivery schedules using a discrete event simulator were performed. The results obtained encourage to implement the decision model in practical application of LPG distribution.

Further research directions: find exact algorithms, completely eliminate stock-out events and reduce ratio number of kilometers driven per ton of gas delivered.

Many Polish and European hauliers are concerned about lack of their assets under- or over-performance. As part of this study some delivery planning and scheduling methods have been verified to increase efficiency of LPG logistics within the entire transport system.

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Marek Karkula
AGH University of Science and Technology, Poland
mkarkula@zarz.agh.edu.pl

Jacek Kaleta
Cracow University of Technology, Poland
jacekkaleta@poczta.onet.pl

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