Mathematical Model of Errors of Odometry and Georeferencing Channels in Visual Correlation Extreme Navigation

Maryna Petrivna Mukhina, Volodymyr Petrovych Kharchenko National Aviation University, Ukraine

The mathematic model of errors in correlation with the extreme navigation system (CENS) is developed basing on odometry and geo-referencing channels. The realization of the model is done in Simulink, and based on regular and random components of additive noise. The results of simulations prove accumulation of errors for odometry errors and its mitigation in case of geo-referencing in periods of correction.

Keywords: correlation extreme navigation, visual odometry, geo-referencing, feature matching, cartographic errors, dead reckoning errors.

1. INTRODUCTION

UAV navigation in GPS-denied environment can possibly rely on low-cost dead reckoning system like inertial navigation system (INS) and therefore needs continuous correction from external sources. Promising variant of such correction is use of visual information available onboard. Realization of correlation extreme principle is based on comparison of current image (or frame) with template geo-referencing image and on finding coordinates of UAS location. Visual data has rich informativity and provides the extraction of such flight and navigation parameters like coordinates, velocity, orientation angles, etc. Their values can be used further in data fusion schemes to minimize the error accumulation in INS. Errors of visual correlation extreme systems may influence significantly the accuracy and efficiency of data fusion and therefore require strong attention and research.

Sources of errors during navigation process by visual observation:

noises caused by technical imperfection of camera (fixed pattern noise, dark current noise, shot noise, amplifier noise and quantization noise [1]) An estimate of noise level can be obtained by noise level function [2] for a given image, which predicts the overall noise variance at a given pixel as a

function of its brightness (a separate function is estimated for each color channel).

- cartographic errors due to inaccuracy of template images, operative changes of environment, use of different reference systems, etc [3].
- measurement method errors because of coordinate system conversion, inaccuracy of matching process [4], etc.
- Among others it is also necessary to take into account the type of used visual navigation: absolute (or geo-referencing) [5] and relative (so called visual odometry [6]).

2. PREVIOUS RESEARCHES ANALYSIS

Random component of visual CENS errors in most researches [7] is represented as additive Gaussian white noise with zero mean or possible as a mix of Gaussians. Constant component is used to describe measurement-method errors to be calibrated during pre-flight procedure.

A geo-referencing system for absolute UAV positioning was developed in [7]. The position reference is expressed as a standard measurement equation, making it easy to incorporate into any sensor fusion framework. The system makes use of environmental classification and rotation invariant template matching, making it robust to variations in the operational environment as well as errors in

the estimated orientation of the vehicle. Any probabilistic classifier can be used together with the proposed geo-referencing system.

The measurement model is available as a 2D look-up table with additive noise. The noise distribution is derived from the classification result, reflecting the uncertainty in the classification.

Modeling and analyzing long-range drift in visual odometry was done in [8]. Drift in visual odometry is represented using an unbounded system model, and its analysis is contains estimating the distance-varying trends, computing a sample function of the residual process, and characterizing the residual as a first-order Gauss-Markov process.

Modeling drift using an unbounded system model in form of a combination of a deterministic part and a first-order Gauss-Markov process is validated. Drift in visual odometry will increase exponentially with the distance traveled. Quantifying drift from a specific algorithm is a more reasonable way than the usual offset ratio method.

3. PROBLEM STATEMENT

The dynamic equations of UAV motion are considered in geodetic coordinate system with further conversion in local tangent plane coordinate system of Earth-North-Up (ENU) type (fig. 1).



Fig. 1. Coordinate systems.

They have the following form:

$$\dot{L} = \frac{V_E}{(R_2 + H)\cos B};$$

$$\dot{B} = \frac{V_N}{R_1 + H};$$

$$\dot{H} = V_{U_1}$$
(1)

where V_N , V_E are northern and eastern components of ground speed (projections on axes of ENU system); *B*, *L* are geodetic coordinates (latitude and longitude); R_1 , R_2 are radiuses of curvature of terrestrial geoid; R_1 is radius of curvature of meridian cross-section by plane *UN*; R_2 is radius of curvature of geoid cross-section by plane *UE*. Radiuses can be found from expressions

$$R_{1} = \frac{a(1-e^{2})}{(1-e^{2}\sin^{2}B)^{\frac{3}{2}}}; \quad R_{2} = \frac{a}{\sqrt{1-e^{2}\sin^{2}B}},$$

where *a* is the major semi-axis of ellipsoid (a = 6378388 m); e is eccentricity of ellipsoid $(e^2 = 6,73 \cdot 10^{-3}); H$ is the flight altitude.

It is possible to simplify the expression with terrestrial radiuses with approximation order 10^{-2} as following:

$$\frac{1}{R_1 + H} \approx \frac{1}{a} \left[1 - e^2 - \frac{H}{a} - \frac{3}{2} e^2 \sin^2 B \right];$$
$$\frac{1}{R_2 + H} \approx \frac{1}{a} \left[1 - \frac{H}{a} - \frac{1}{2} e^2 \sin^2 B \right].$$

4. PROPOSED MODELS OF ERRORS

Generalized representation of mathematical model of visual CENS errors in odometry channel is the following:

$$V_{N}^{VCENS} = V_{N} + \Delta_{V_{N}^{VCENS}} + \xi_{V_{N}^{VCENS}},$$

$$V_{E}^{VCENS} = V_{E} + \Delta_{V_{E}^{VCENS}} + \xi_{V_{E}^{VCENS}},$$
(2)

where V_N , V_E are true values of northern and eastern components of ground speed; V_N^{VCENS} , V_E^{VCENS} are components of ground speed measured by visual CENS in odometry channel; $\Delta_{V_N^{VCENS}}$, $\Delta_{V_E^{VCENS}}$ are constant or slowly varied components of errors in velocity, caused by inaccurate camera orientation; $\xi_{V_N^{VCENS}}$, $\xi_{V_E^{VCENS}}$ are random or fluctuating noises caused by camera noises, mismatching of feature points, etc.

Generalized representation of mathematical model of visual CENS errors in absolute positioning (geo-referencing) channel is the following:

$$B_{VCENS} = B + \xi_B,$$

$$L_{VCENS} = L + \xi_L,$$
(3)

where *B*, *L* are true values of latitude and longitude; B_{VCENS} , L_{VCENS} are coordinates determined by geo-referencing; ξ_B , ξ_L are random noises caused by false matching.

5. SIMULATION RESULTS

Simulation of errors of visual CENS in odometry channel is done by assuming the flight with constant ground speed components: $V_E = 100$ m/sec; $V_N = 5$ m/sec; and slowly varying altitude

of flight as V_U by sine law with amplitude ±50 m. To compare accuracy the reference (ideal) navigation system is introduced parallel to CENS (Fig. 2).

Reference system (Fig. 3) uses ideal signals of ground speed components and realizes the model dynamics given in (1).

Parameters of HF and LF errors in each measured component of ground speed are selected as following: constant drift is of 1% of valid signal; noise power of HF error is 5% of valid signal (Fig. 4).



Fig. 2. General block diagram in Simulink to simulate errors of visual CENS in comparison with some reference navigation system.



Fig. 3. Reference navigation system.



Fig. 4. Odometry channel of visual CENS and additive noise according to (2).

The simulation results are shown in Fig. 5. Errors in coordinates are accumulated over time which is true since odometry in CENS is realized by dead reckoning principle.



Fig. 5. Errors in latitude (a) and longitude (b) by visual odometry channel.

Now let us introduce absolute positioning from geo-referencing by matching the current image and satellite images. It can be assumed that there is no error in recognition, and matching provides the true values of coordinates (Fig. 6). The condition of correction is set beforehand in time period 450-500 sec, where the availability of reliable landmarks to be recognized is assumed. The correction is done in the form of increments in latitude and longitude to be applied to integrators, since pure replacement of true coordinates will not change the initial conditions of integration and will not eliminate the accumulation of errors after the period of reliable recognition.

The results of simulation are shown in Fig. 7. As it can be seen during the period of reliable correction from geo-referencing the errors in coordinates are minimized up to zero level (in case of absence of matching error). Then the rate of error accumulation returns to its original value until the next correction period.



Fig. 6. Channel of geo-referencing by satellite imaginary (no error).



Fig. 7. Errors of CENS odometry channel with correction by absolute geo-referencing positioning.

6. CONCLUSIONS

The proposed mathematical model of errors of CENS satisfies the experimental results of accuracy characteristics of odometry channel. The accumulation of errors can be minimized in period of correction by geo-referencing channel.

REFERENCES

- Tsin, Y., Ramesh, V., and Kanade, T. (2001). Statistical calibration of CCD imaging process. In Eighth International Conference on Computer Vision (ICCV 2001), pp. 480–487, Vancouver, Canada
- [2] Grossberg, M. D. and Nayar, S. K. (2004). Modeling the space of camera response functions.

IEEE Transactions on Pattern Analysis and Machine Intelligence, 26(10):1272–1282.

- [3] Goodchild M. F., Gopal S. (ed.). The accuracy of spatial databases. – CRC Press, 1989. –193 p.
- [4] V. Kharchenko, M. Mukhina Correlation-extreme visual navigation of unmanned aircraft systems based on Speed-Up Robust Features // Aviation. No 2, Volume 18, 2014.
- [5] Conte G., Doherty P. Vision-based unmanned aerial vehicle navigation using geo-referenced information //EURASIP Journal on Advances in Signal Processing. – 2009. – P. 10.
- [6] Caballero F. et al. Vision-based odometry and SLAM for medium and high altitude flying UAVs //Unmanned Aircraft Systems. – Springer Netherlands, 2009. – P. 137-161.
- [7] Lindsten F. et al. Geo-referencing for UAV navigation using environmental classification

//Robotics and Automation (ICRA), 2010 IEEE International Conference on. – IEEE, 2010. – P. 1420-1425.

[8] Jiang R., Klette R., Wang S. Modeling of unbounded long-range drift in visual odometry //Image and Video Technology (PSIVT), 2010 Fourth Pacific-Rim Symposium on. – IEEE, 2010. – P. 121-126.

> Date submitted: 2015-06-11 Date accepted for publishing: 2017-04-01

> > Maryna Petrivna Mukhina National Aviation University, Ukraine m_mukhina@inbox.ru

> > Volodymyr Petrovych Kharchenko National Aviation University, Ukraine kharch@nau.edu.ua