Some Aspects of the Application of Genetic Algorithm for Solving the Assignment Problem of Tasks to Resources in a Transport Company

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The article defines the assignment problem of tasks to resources in a transport company. The paper describes mathematical model of a transport system taking into account the assignment of vehicles to the tasks. It also provides stages of creation of the genetic algorithm for solving the assignment problem in the transport company.

Keywords: assignment problem, genetic algorithm, optimization.

1. INTRODUCTION

We live in the era of high competition on the transport market. Customers' requirements impose the way of working of transport companies. The main factors which influence the quality of the resource tasks are: speed of order execution, deadline and punctuality. All these factors determine the quality of transport services and thus build reputation of a company. One of the main issues of the company, that affect quality of service, is the problem of assignment of tasks. The classic problem of allocation widely described in the literature is the linear discrete optimization problem[4]. The assignment problem of tasks to resources in the transport company includes also additional time constraints and payload limitations. In the literature, the problem of allocation of tasks to resources in a transport company refers to setting timetable for drivers [5][8], i.e. selection of drivers to the particular tasks on the working day.

In this paper the problem of allocation refers to routing of vehicles. The route consists of a set of implemented tasks, and the problem of allocation is associated with routing of a vehicle at the end of a current task to the next one in a way that would ensure the completion of the entire route at minimal expense.

The issue of allocation of tasks to resources in a transport company is a complex decision problem. There are many different factors which should be taken into consideration. These are, among others: a task duration, daily working time of drivers, task realization expense, number of the means of transport of a specific type, etc. The nature of the business and the type of tasks enhance further the complexity of a problem.

In order to solve the problem of allocation of tasks in a transport company, a genetic algorithm was constructed which determines the order of tasks implemented by vehicles on a transport route. The order essentially influences the length of the entire route and the value of the objective function.

2. DEFINITION OF THE ALLOCATION PROBLEM IN A TRANSPORT COMPANY

The research problem applies to a transport company involved in goods transport. The transport task in a company can be defined as an acquisition of a product from place of loading and its transportation to the point of unloading. The limitations of the transport tasks are imposed in a form of a time window for reception and delivery of a cargo while an exact quantity of transported goods is provided. The problem of allocation depends on limitations of working and driving time of drivers. During the implementation of a transport task, the moment of allocation occurs at the point, where a vehicle at the end of a task is presented with two options: either to begin an implementation of the next task or to return to base. The beginning of the next task depends on the limitations of drivers'
working and driving time, vehicle capacity and time window of delivery and reception. The problem of allocation in the transport company refers to routing of a vehicle at the end of the implementation of the current task to the next one. The whole generated route length has to be minimalized. The allocation to the next task is not always possible due to realization of overlapping tasks (tasks are executed at the same time), if hour of the end of the current task exceeds the starting hour of an unrealized task. One should also take into consideration the travel time between tasks. It affects possibility of the allocation appearance. Tasks can start and end at the same time, and in such a case the allocation to the next task is impossible. Various combinations of time decide on the possibility of the allocation to the next task. Therefore the realization of such a task is not always easy.

Wasteful and irrational would be for a single vehicle to perform only a single task concluded with a return to base. The actual aim is therefore to create multitasking routes where the allocations decide on the length and hence the cost of implementation of the entire route (Figure 1).

Additional decision variables have been introduced and they define the connection: the base–task and the task–base in order to determine the minimal cost of implementing all transport routes. The assumption of the model of allocation of tasks to resources is that all tasks are commissioned to be implemented on the same working day and the national transport is taken into consideration accordingly.

In order to present the function of criterion and constraints, following data has been specified [3]:

- \( W^Z \) – the set of points of loading, \( W^Z = \{1, ..., i, ..., W^Z\} \), \( i \) - other element of the set of \( W^Z \),
- \( W^W \) – the set of points of unloading, \( W^W = \{1, ..., j, ..., W^W\} \), \( j \) – other element of the set of \( W^W \),
- \( W^B \) – the set of the transport base numbers, \( W^B = \{1, ..., b, ..., W^B\} \), \( b \) – other element of the set of \( W^B \),
- \( P \) – the set of vehicles' numbers, \( P = \{1, ..., p, ..., P\} \), \( p \) – another vehicle of the set of \( P \),
- \( N \) – the set of drivers' numbers, \( N = \{1, ..., n, ..., N\} \), \( n \) – another driver of the set of \( N \),
- \( W = [w(j, i)] \) – the matrix of the distance between \( j \) - this point unloading and \( i \) - this point of loading,

![Fig. 1. Example of transport routes taking into account the allocation of the vehicle to the tasks in the transport company: PZ – the point of loading, PW – the point of unloading. Source: compiled by author.](image-url)
• $WZ = [wz(i,j)]$ – the matrix of the distance between $i$ - this point of loading and $j$ - this point unloading,

• $BZ = [bz(b,i)]$ – the matrix of the distance between $b$ - this base and $i$ - this point of loading,

• $WB = [wb(j,b)]$ – the matrix of the distance between $j$ - this point of unloading and $b$ - this base,

• $T1 = [t1(p,n(i,j))]$ – the matrix of travel times between $i$ - this point of loading and $j$ - this point of unloading for $p$ – this vehicle and $n$ - this driver,

• $T2 = [t2(p,n(j,i))]$ – the matrix of travel times between $j$ - this point of unloading and $i$ - this point of loading for $p$ – this vehicle and $n$ - this driver,

• $T3 = [t3(p,n(b,i))]$ - the matrix of travel times between $b$ - this base and $i$ - this point of loading for $p$ – this vehicle and $n$ - this driver,

• $T4 = [t4(p,n(j,b))]$ – the matrix of travel times between $j$ - this point of unloading and $b$ - this base for $p$ – this vehicle and $n$ - this driver,

• $T7 = [t7(p,n,i)]$ - the vector of times of loading of a vehicle $i$ - of this point of loading for $p$ – this vehicle and $n$ - this driver,

• $T8 = [t8(p,n,j)]$ – the vector of times of unloading of a vehicle $j$ - of this point of unloading for $p$ – this vehicle and $n$ - this driver,

• $T5 = [t5(p,n,i)]$ – hour of departure $p$ – of this vehicle and $n$ - of this driver from $i$ - this point of loading,

• $T6 = [t6(p,n,j)]$ – hour of departure $p$ – of this vehicle and $n$ - of this driver from $j$ - this point of unloading,

• $T^\text{odp}$ – statutory resting time on the route,

• $(a_i, b_i)$ – the window of time $i$ - of this point of loading,

• $k^p$ - the unit cost of usage $p$ – of this vehicle per unit of road,

• $k^n$ - the unit cost of working time $n$ - of this driver,

• $\theta(i)$ – volume of the load $i$ - of this point of loading,

• $\varphi 1(p)$ - payload $p$ – of this vehicle,

• $\tau^\text{dop}$ - the permitted driving time,

• $\tau^\text{dop}$ - the permitted working time of driver,

• $xzw(p,n,(i,j))$ – binary matrix defining the road between $i$ - this point of loading and $j$ - this point of unloading.

A binary decision variable of allocation $x(p, n, (j, i))$ must be determined for this data to determine the road length between the point of unloading of the current task and the point of loading of the next task, and auxiliary variables creating the transport route: $xbz(p, n, (b, i))$ a variable determining the road between the base and the point of loading of the first task which is realized on the route and $xbw(p, n, (j, b))$ variable determining the road between the point of unloading of the last task realized on the route and the base where:

\[
x(p, n, (j, i)) = \begin{cases} 
1 & \text{the route from (j) to(i)} \\
0 & \text{otherwise}
\end{cases}
\]

\[
xbz(p, n, (b, i)) = \begin{cases} 
1 & \text{the route from (b) to(i)} \\
0 & \text{otherwise}
\end{cases}
\]

\[
xbw(p, n, (j, b)) = \begin{cases} 
1 & \text{the route from (j) to(b)} \\
0 & \text{otherwise}
\end{cases}
\]

for which the function of criterion takes the value:
Constraints take the form of:

- The limit of allocation to the following tasks:

\[ \forall p \in P, i \in W^Z, j \in W^W, n \in N \]

\[ t_6(p, n, i) + x(p, n, (j, i)) \cdot t_2(p, n, (j, i)) + T_{dop} \leq b_i \]

(5)

The allocation to the next tasks depends on the departure hour from the unloading point, the travel time to the next task and the possibly of rest for the driver on the route. The hour received should be contained within the time window corresponding to the another point of loading of the next task.

- The limit of driving time of a driver:

\[ \forall p \in P, i \in W^Z, j \in W^W, n \in N, b \in W^B \]

\[ x_{bz}(p, n, (b, i)) \cdot t_3(p, n, (b, i)) + x_{bz}(p, n, (b, i)) \cdot x_{zw}(p, n, (i, j)) \cdot t_1(p, n, (i, j)) \]

\[ + \sum_{i \in W^Z} \sum_{j \in W^W} \sum_{n \in N} x(p, n, (j, i)) \cdot t_2(p, n, (j, i)) \cdot t_4(p, n, (j, i)) \leq T_{dop} \]

(6)

The time of driving of a driver is the travel time from the base to the point of loading of the first task, the amount of driving time in the next tasks, the amount of time to arrive to the next tasks and time spent to return to base. The eventual working time of the driver must be less than or equal to the permitted time.

- The limit of the working time of a driver:

\[ \forall p \in P, i \in W^Z, j \in W^W, n \in N, b \in W^B \]

\[ x_{b}(p, n, (b, i)) \cdot t_3(p, n, (b, i)) + x_{b}(p, n, (b, i)) \cdot x_{zw}(p, n, (i, j)) \cdot t_1(p, n, (i, j)) \]

\[ + \sum_{i \in W^Z} \sum_{j \in W^W} \sum_{n \in N} x(p, n, (j, i)) \cdot t_2(p, n, (j, i)) \cdot t_4(p, n, (j, i)) \leq T_{dop} \]
The time of working of driver is the time to arrive to the tasks, time of waiting for loading and unloading of cargo, time of implementation of tasks and time of returning to base. The eventual working time of the driver must be less than or equal to the permitted working time.

\[
x_{bw}(p, n, (b, i)) \cdot t_3(p, n, (b, i)) + x_{bw}(p, n, (b, i)) \cdot t_7(p, n, i) + \sum_{i \in W^Z} \sum_{j \in W^W} x(p, n, (j, i)) \cdot t_7(p, n, i)
+ x_{bw}(p, n, (b, i)) \cdot x_{zw}(p, n, (i, j)) \cdot t_1(p, n, (i, j))
+ \sum_{i \in W^Z} \sum_{j \in W^W} [x(p, n, (j, i)) \cdot x_{zw}(p, n, (i, j)) \cdot t_1(p, n, (i, j))]
+ \sum_{i \in W^Z} \sum_{j \in W^W} x(p, n, (i, j)) \cdot t_2(p, n, (i, j)) + \sum_{i \in W^Z} \sum_{j \in W^W} x(p, n, (j, i)) \cdot t_8(p, n, i) + x_{wb}(p, n, (j, b)) \cdot t_4(p, n, (j, b)) \leq T^{dep}
\]

4. STAGES OF A GENETIC ALGORITHM OF THE ALLOCATION PROBLEM

Genetic algorithms are algorithms whose activities are based on the mechanisms of natural selection and heredity. They are used as a functional and practical optimization tool [1]. The main advantages of the genetic algorithms over other methods of optimization are as follows: conducting the search for the optimum not from a single point in the plane of search but from several points, established by the relevant population of individuals, and reliance on the information determined by the objective function and not derivatives. Basing on the values of an objective function is a valuable advantage of genetic algorithms. The objective function provides us with the value by which a genetic algorithm finds the acceptable and satisfactory solution from the point of view of the problem. It should be noted that the genetic algorithm is a heuristics. Methods of this type give a near-optimum solution, can find the optimal solution but often confine themselves to the optimal solution in the local area of search. Despite this inconvenience genetic algorithms are successfully used in optimization problems.

The basic elements of a genetic algorithm are the following:

- A chromosome – a sequence of data, such as a string of bits, integer numbers, matrix of bits and of total numbers.
- initial population – a collection of random initial chromosomes.
- function of adaptation – the objective function for determining value of a single chromosome.
- a pool of parent – population of individuals with the best functions of adaptation.
- A generation –a newly established population of individuals including all genetics.

The principle of operation of a classical genetic algorithm consists of choosing randomly initial population, and then repeating three basic operations: selection, crossover and mutation, until the moment predetermined by an algorithm or until absence of any changes in the improvement of permissible solutions. Researchers of genetic algorithm have used a variety selection methods [1][6], for example: random selection with repetitions, without repetitions. Classical methods of crossover, based on a crossover of individuals, represented as binary or numerical strings, are precisely described. The operation of mutation depends on the kind of problem and has not been clearly defined.

In this paper the basic genetic operators were used, suitable for the structure of input data. As an operator of selection a roulette method was taken, while as an operator of crossover an operator appropriate for the crossover of strings of integral numbers (PMX). The mutation was made on the basis of swapping two values of a gene in the chromosome. Additionally the inversion operator was introduced as the operator of change of the setting of strings in the chromosome.

The genetic algorithm of allocation problem in the transport company consists of the following steps:

- Step 1 – Determining a structure of the data processed by the algorithm.
• Step 2 – Selection of chromosomes dependent on the function of adaptation.
• Step 3 – Crossover of chromosomes selected randomly out of the pool of parent.
• Step 4 – Mutation of chromosomes.
• Step 5 – Inversion.

All the steps of genetic algorithm are repeated until the stop condition is achieved. The stop condition in the allocation problem is a predetermined number of generations (iterations).

4.1 General evaluation of the acoustic microclimate

The basic structure of the input data for the classical genetic algorithm is a zero–one sequence. In the problem of allocation in a transport company, the chromosome is represented by the string of natural numbers. Such a structure of the input data successfully works out in similar cases where the use of zero–one strings significantly hinders the operation of the genetic algorithm, e.g. in the traveling salesman problem [7]. The genetic algorithm does not work directly on the decision variables of the function of criterion, but on the encoded forms of these variables. In order to encode the variables of the function of criterion in an appropriate structure and create the chromosome as a representative of the admissible solution, the problem of allocation must be defined as the appointment of the suitable permutation of tasks and bases, so that their location would generate the minimum value of the function of criterion, which in our case is the minimum cost of all tasks. Time constraints make it not always possible to implement all the tasks in a single transport route, thus there may be several routes realized by different vehicles. Therefore a chromosome consists of the bases which provide for the occurrence of more than one transport route. The task of the genetic algorithm is to find the best set of tasks and bases by optimizing function of adaptation. The structure of data suitable for processing by the genetic algorithm can be defined as a string consisting of the implemented zd tasks and k bases. The total length of the chromosome for the occurrence of one base is 2zd + 1 genes. For the \( k > 1 \) the pattern for the maximum length of the chromosome takes the form of 3zd. The maximum length of the chromosome is imposed by the situation in which a single vehicle performs only one task and returns to base, thus for \( k = 1 \) space between tasks considers a single gene identifying the base which is both a starting and finishing point of realized routes (Fig 2b). For \( k > 1 \) space between tasks considers two genes identifying the location of the base, one of which is an ending point of the realized route, while the second one a starting point of the next route (Fig. 2c).

Each chromosome is a representative of transport route or routes. Number of the routes contained in the chromosome depends on the constraints which determine whether all the tasks are realized in one or many routes. Each route starts and ends in a base. Therefore the starting and ending chromosome gene is always the base. In order to distinguish the bases in the chromosome, bases must be marked by indexes. At the beginning and the ending of each chromosome, there is a one-index base. The second index is introduced only to distinguish the location of bases within the chromosome. Such identification of the base is done in order to implement the process of the crossover. Physically the base with a single index has the same meaning that the base with a double index e.g.: the base with number \( 0_1 = 0_{11} = 0_{12} = 0_{13} = 0_{14} = 0_{15} \).

![Fig. 2. The determining of the maximum size of a chromosome for zd = 6 tasks, a) numbers of tasks for implementation. b) the maximum chromosome providing for \( k = 1 \) base, c) the maximum chromosome providing for \( k = 2 \) base, d) the random set of genes in the chromosome for \( k = 1 \) base. Source: compiled by author.](image-url)
4.2 Selection of chromosomes

The operation of reproduction (selection) consists of duplication of chromosomes depending on the function of adaptation. The chromosomes with the higher function of adaptation are more likely to introduce their own copy to the next generation according to the analogy to Darwinian principle of natural selection where the fittest individuals survive. There are many algorithms performing the operation of selection such as a tournament selection and a roulette selection where all the chromosomes are evaluated in each generation according to their function of adaptation and selected for the next population. In selection process the roulette method was used based on the selection of a new population according to the probability distribution defined on the values of the function of adaptation.

The selection process consists of the following stages:

- Calculation of the function of adaptation for a single chromosome $-F_l$.
- Calculation of the total population of adaptation, where $L$ is defined as the cardinality of the population $L = \{1, ..., l, ..., L\}$:
  \[
  F_{tot} = \sum_{l=1}^{L} F_l
  \]  
  (9)

- Calculation of the probability of the selection $l$ of the chromosome:
  \[
  p_l = \frac{F_l}{F_{tot}}
  \]  
  (10)

- Calculation of the distribution $l$ of the chromosome:
  \[
  q_l = \sum_{s=1}^{l} p_s
  \]  
  (11)

Choosing the chromosome to the next generation consists of the random selection of the number $r$ from the range of $[0,1]$. We choose $l$ – the chromosome with the value of distribution $q_l$ while the relationship $q_{l-1} < r \leq q_{l}$ is fulfilled.

4.3 Crossover of chromosomes

In the crossover operation an operator which works on the structure of the numeric strings was used, called PMX (partially matched crossover). The PMX crossover is one of the types of crossovers used in problems where the chromosomes can be shown as the permutations of tasks e.g.: the traveling salesman problem [7]. The PMX crossover is a random selection of two chromosomes in pairs, random selection of two points of crossover and exchange of genes shown by series created from these points of crossover. The bases are marked by indexes in order to distinguish them in the process of crossover. In the process of crossover genes from one chromosome are assigned to genes from the other one. It is possible that as a result of the crossover the first gen of a chromosome or the last one change the meaning, the base is swapped with the task. In order to avoid such a situation the starting and ending base in the chromosomes should be distinguished from the bases inside the chromosome through the introduction of one- and two-element indexes. The PMX crossover is shown in Fig.3.

Fig. 3. The PMX crossover a) the chromosomes used to the crossover b) a swap of genes c) the chromosomes after crossover. Source: compiled by author.

4.4 Mutation and inversion

A mutation is a swap of place of two randomly selected tasks. The principle of the mutation is shown in Fig.4.

Fig.4. The use of a mutation operator. Source: compiled by author.
To change the arrangement of the order of tasks a simple inversion operator was used. The initial stage of inversion is a random selection of two points in the chromosome. These points create the string which needs to be reversed. The principle of the inversion is shown in Fig. 5.

Mutation and inversion operators are designed to further enhance a search space of the genetic algorithms.

![Fig. 5. The principle of the inversion. Source: compiled by author](image)

5. SUMMARY

The aim of this study was to determine initial assumptions and ways of realization of the algorithm solving the problem of allocation of tasks to resources in the transport company. Genetic algorithm steps described in this paper include basic methods and operators appropriate for the assumed structure of data, such as the roulette method or PMX crossover. Diversity of genetic operators gives a possibility of experimentation with the assignment problem and makes a basis for a further research on this topic. The further research related to the problem will consist of building of an algorithm which would solve the assignment problem in a transport company and implementing it for the computer simulation.

BIBLIOGRAPHY


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