Modelling of Curvature of the Railway Track Geometrical Layout Using Particle Swarm Optimization

Katarzyna Małgorzata Palikowska
Gdansk University of Technology, Poland

A method of railway track geometrical layout design, based on an application of cubic C-Bezier curves for describing the layout curvature is presented in the article. The control points of a cubic C-Bezier curve are obtained in an optimization process carried out using Particle Swarm Optimization algorithm. The optimization criteria are based on the evaluation of the dynamic interactions and satisfaction of geometrical design requirements.

Keywords: Particle Swarm Optimization algorithm, cubic C-Bezier curve, curvature of the railway track layout, dynamic interactions, transition curve.

1. INTRODUCTION

Design of railway track geometrical layout consists of joining of certain characteristic points of the route using straight lines and arcs of the fixed and variable curvatures. At joining points, as a result of horizontal curvature changes, increased dynamic interactions in the vehicle-route system occur [12]. In the process of curvature modelling the designer tries to ensure smooth curvature changes, meeting specific conditions [10], through appropriate configuration of transition curves, providing the most advantageous dynamic properties of the layout.

Investigation and evaluation of transition curves are still current, as evidenced by numerous publications on these issues. Two main trends can be distinguished in research aimed at extending the available options for modelling of transition curves: a direct shaping of the coordinates of the transition curve and an indirect one, achieved by modelling the curvature. In the papers [3, 6, 7], enrolling in the coordinates shaping trend, algorithms of construction of Bezier curves as transition curves in highway and railway layouts design have been presented. The advantageous application of cubic C-Bezier curves, presented in [3, 19] and Pythagorean hodograph quintic Bezier curves, presented in [6, 7], as transition curves joining two circular arcs with reverse curvatures (*S-shaped* transition) and consistent curvatures (*C-shaped* and *C-oval* transitions) has been confirmed in [12, 13] by successfully using considered curves in exemplary railway geometrical layouts.

In the papers [12, 13] dynamic properties of Bezier curves have been evaluated in comparison to dynamic properties of K^0 curve (a transition curve with linear curvature changes) and K^1 curve (a transition curve with nonlinear curvature changes described by third degree polynomial of variable l standing for curve length), obtained by applying a universal method of curvature modelling.

This paper presents a new method of curvature modelling – curvature of railway track geometrical layout is described by a cubic C-Bezier curve [19] and shaped by appropriate location of the curve control points. The coordinates of the control points are obtained in an optimization process carried out using Particle Swarm Optimization (PSO) algorithm with the optimization criteria based on the evaluation of the dynamic interactions in the vehicle-route system and the satisfaction of the geometrical constraints.

In contrast to papers [3, 12, 13] in the presented method a C-Bezier curve has been applied for describing curvature of the railway geometrical layout instead of representing direct coordinates of the transition curve. This fundamental difference requires additional effort to satisfy certain geometrical constraints imposed on each transition curve [10] but provides some promising opportunities due to direct curvature shaping. Taking into account current high speed challenges, an appropriate curvature shape (especially at joining points of segment with different curvature) is crucial from dynamic interactions point of view.

2. CURVATURE MODELLING

The regulations for highway and railway geometrical design impose a set of requirements on transition curves [10]. Satisfaction of imposed requirements is tightly connected with ensuring appropriate shape of curvature *k(l)* and its first and second derivatives, *k'(l)* and *k''(l)* respectively. Numerous papers have been devoted to modelling curvature of the highway [2, 17, 18] and railway [12, 13, 15] layouts.

The universal method of curvature modelling, presented in [12], is based on the assumption that curvature function $k(l)$ is a solution of the following differential equation:

$$k^{(m)}(l) = f\left[l, k, k', \dots, k^{(m-1)}\right] \tag{1}$$

where: l – arc length from the origin point of the curve to the chosen point along the curve, $k'(l)$, ... , $k^{(m-1)}(l)$, $k^{(m)}(l)$ – successive derivatives of the curvature function $k(l)$,

with boundary conditions at the origin point (for $l = 0$) and at the end (for $l = l_k$) of transition curve. The boundary conditions are:

$$k^{(i)}(0^+) = \begin{cases} k_1 & dla \quad i = 0 \\ 0 & dla \quad i = 1, 2, \dots, n1 \end{cases} \tag{2}$$

$$k^{(j)}(l_k^-) = \begin{cases} k_2 & dla \quad j = 0 \\ 0 & dla \quad j = 1, 2, \dots, n2 \end{cases} \tag{3}$$

The order of the differential equation (1) is $m = n_1 + n_2 + 2$, while the obtained curvature function $k(l)$ is of class C^n in the range $\langle 0, l_k \rangle$, where $n = \min(n_1, n_2)$. The method enables joining the route segments of different curvature, including two circular arcs with consistent and reverse curvatures (*C-shaped, C-oval and S-shaped* transitions), using transition curves with linear and nonlinear curvature changes, depending on the assumed order of differential equation (1) and a set of boundary conditions (2)-(3). Presented in [12] K^o curve, has been obtained as a solution to the following second order differential equation:

$$k''(l) = 0 \tag{4}$$

with boundary conditions:

$$\begin{cases} k(0^+) = k_1 \\ k(l_k^-) = k_2 \end{cases} \tag{5}$$

The obtained curvature – K^o curve, a linear function of curve length *l* joining two circular arcs:

$$k(l) = k_1 + \frac{1}{l_k}(k_2 - k_1)l \tag{6}$$

is used in this paper as a reference curvature in a comparative analysis of dynamic properties of transition curves obtained by the method presented in this paper (Section 5).

After obtaining curvature $k(l)$, the primary task is to determine the coordinates of the transition curve in a Cartesian coordinate system x, y. It is assumed that the origin point O of the system is located at the end point of the curve with the curvature k_1 and x-axis is tangent to the curve at that point (Fig. 1). The equation of the desired transition can be written in a parametric form:

$$x(l) = \int \cos \Theta(l) dl \tag{7}$$

$$y(l) = \int \sin \Theta(l) dl \tag{8}$$

The parameter l represents a current position of the chosen point along the curve length. The function of slope of the tangent $\Theta(l)$ is defined by the formula:

$$\Theta(l) = \int k(l) dl \tag{9}$$

In the study [15] an application of evolution programming to modelling curvature of railway layout has been presented. The curvature monotonicity condition has been ensured a priori by encoding of the curvature ordinates increments. A classical set of genetic operators has been widen: apart from mutation and crossover has included inversion, deletion, duplication and filtering operators, ensuring desirable curvature changes during the optimization process.

3. THE PROPOSED METHOD

This paper presents an idea of describing a curvature by a cubic C-Bezier curve [3, 19], defined parametrically as follows:

$$\boldsymbol{P}(t)=\frac{2}{\pi-2}\begin{bmatrix}\sin t\\ \cos t\\ t\\ 1\end{bmatrix}^T\begin{bmatrix}0 & \frac{2-\pi}{4-\pi} & \frac{2}{4-\pi} & -1\\ -1 & \frac{2}{4-\pi} & \frac{2-\pi}{4-\pi} & 0\\ -1 & \frac{2}{4-\pi} & \frac{-2}{4-\pi} & 1\\ \frac{\pi}{2} & \frac{-2}{4-\pi} & \frac{\pi-2}{4-\pi} & 0\end{bmatrix}\begin{pmatrix}\boldsymbol{P_0}\\ \boldsymbol{P_1}\\ \boldsymbol{P_2}\\ \boldsymbol{P_3}\end{pmatrix} \quad (10)$$

for the parameter t in the interval $0 \le t \le \frac{\pi}{2}$. Among the control points $\{\boldsymbol{P_i}\}_{i=0}^{3}$ of Bezier curve, the points $\boldsymbol{P_0}$ and $\boldsymbol{P_3}$ are at the same time Bezier nodes and the tangency points of the curve and the consistent arcs ($R_1 > R_2$) with curvatures k_1 and k_2 (Fig. 2).

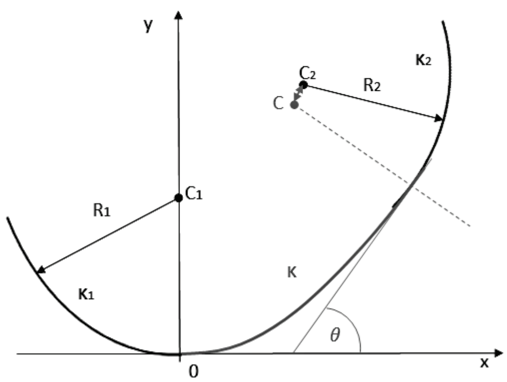

Fig. 1. Transition curve joining two circular arcs with consistent curvatures

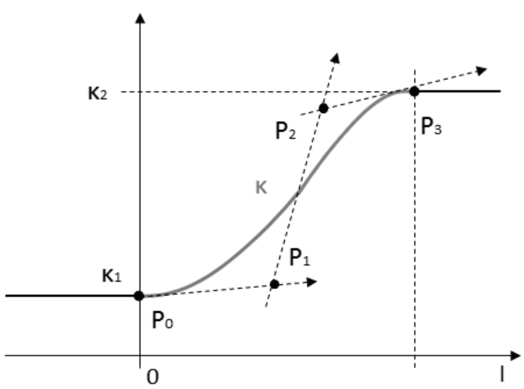

Fig. 2. Curvature of the transition curve joining two circular arcs with consistent curvatures

The geometrical layout of a transition curve joining two circular arcs with consistent curvatures is presented in Figure 1. It is assumed that the locations of the arc centres C_1 and C_2 are given as the requirements of a design task. The location of the point C, which is the centre of the circular arc with a radius perpendicular to the curve tangent at the curve end point, depends on the curvature, from which using formulas (7) – (9) transition curve coordinates x, y are determined.

The first derivative of the curvature (10) is given as follows:

$$\boldsymbol{P}'(t)=\frac{2}{\pi-2}[(1-\sin t)(\boldsymbol{P_1}-\boldsymbol{P_0})+(1-\cos t)(\boldsymbol{P_3}-\boldsymbol{P_2})]+\frac{2}{4-\pi}(\cos t+\sin t-1)(\boldsymbol{P_2}-\boldsymbol{P_1}) \quad (11)$$

The second derivative of the curvature (10):

$$\boldsymbol{P}''(t)=\frac{2}{\pi-2}\left[\left(\frac{\pi-2}{4-\pi}(\boldsymbol{P_2}-\boldsymbol{P_1})-(\boldsymbol{P_1}-\boldsymbol{P_0})\right)\cos t+\left(\boldsymbol{P_3}-\boldsymbol{P_2}-\frac{\pi-2}{4-\pi}(\boldsymbol{P_2}-\boldsymbol{P_1})\sin t\right)\right] \quad (12)$$

In the paper [3] a C-Bezier curve (10) has been applied for describing a transition curve Cartesian coordinates x, y and an algorithm for determination of the curve control points $\{\boldsymbol{P_i}\}_{i=0}^{3}$ ensuring the satisfaction of the geometrical constraints has been presented. In this paper a C-Bezier curve (10) describes the curvature of the transition curve while its control points $\{\boldsymbol{P_i}\}_{i=0}^{3}$, uniquely defining the curvature, are obtained in the optimization process presented in Section 4. Particle Swarm Optimization algorithm [9] with the optimization criterion based on the dynamic properties evaluation has been applied for determination of the control points coordinates. The effectiveness of application of PSO algorithm for determining the control points of NURBS (*Non-Uniform Rational B-Spline curves*) curve has been proved in [5].

4. OPTIMIZATION PROCESS

4.1 Optimization criteria

Minimization of dynamic interaction in the vehicle-route system was an essential criterion in the numerous papers devoted to the curvature modelling [2, 12, 13, 15, 17]. Evaluation of the dynamic properties has been carried out using various dynamic models and methods. The most common method of transition curves evaluation is based on analysis of the Lateral Change of Acceleration (LCA) diagrams [1, 2, 17].

The optimization criteria applied in this paper are based on the model and method of the dynamic interactions evaluation presented in [11, 15]. The essential element of the dynamic interactions analysis is the determination of the oscillations function $X(t)$ and the resultant acceleration of the oscillating motion $X''(t)$ in the areas of horizontal curvature changes. The changes of horizontal curvature are a forcing factor of the lateral oscillations and the function of lateral unbalanced acceleration $a(l)$ along the transition curve, as it is proved in [11], results directly from a function of curvature $k(l)$. In the study [15] a numerical method for the determination of the function of oscillations *X(t)* has been presented using a simple forced oscillations model of a mass with a spring and a dumper. The second-order differential equation describing the forced oscillations is solved by Störmer method [4] through a transformation into a recursive equation (13):

$$X_{i+1} = \frac{h^2 a_i - X_i\left(\dfrac{h^2\omega^2}{1-D^2} - 2\right) - X_{i-1}\left(1 - \dfrac{D\omega h}{\sqrt{1-D^2}}\right)}{1 + \dfrac{D\omega h}{\sqrt{1-D^2}}} \quad (13)$$

where:

X_{i+1} – output sample of oscillations,
h – size of the discretization step,
a_i – discretized i-sample of the lateral unbalanced acceleration *a(l)* resulting directly from the curvature *k(l)*,
ω – free oscillation frequency,
D – Lehr's damping coefficient [11].

This method has been used in the presented optimization process as an essential element of the optimization criterion. The optimization criterion based on dynamic analysis consists of the determination of the oscillations function $X(t)$, the resultant acceleration of the oscillating motion $X''(t)$ and the evaluation these properties in terms of a real number (i.e. criterion value). The criterion value plays only a comparative role, allowing to create a transition curves rank list and steering the optimization process towards curvatures with better dynamic properties.

In order to prove the usefulness of the dynamic criterion, the evaluation of selected transition curves: clothoid curve, Bloss curve, sinusoidal curve, Tari-1 curve [17], G1 and G2 curves [18] has been carried out. All mentioned curves have been applied as a transition curve of the length l=100 m, joining a straight line with a circular arc (with radius R=500 m), on which the unbalanced centrifugal acceleration a_{max}=0,6 m/s^2. The curvature formulas of the transition curves, that are evaluated, have been presented in Table 1.

Table 1. Curvature formulas for the evaluated curves

Curve type	Curvature formula
Clothoid curve	$k(l) = \frac{l}{RL}$
Bloss curve	$k(l) = \frac{1}{R}\left(\frac{3l^2}{L^2} - \frac{2l^3}{L^3}\right)$
Sinusoidal curve	$k(l) = \frac{1}{R}\left(\frac{l}{L} - \frac{1}{2\pi}\sin\frac{2\pi l}{L}\right)$
Tari-1 curve [17]	$k(l) = \frac{l^3}{RL^3}\left(\frac{6l^2}{L^2} - \frac{15l}{L} + 10\right)$
G1 curve [18]	$k(l) = \frac{l^2}{RL^2}$
G2 curve [18]	$k(l) = \frac{1}{R}\sqrt{\frac{l}{L}}$

The curvatures of the evaluated curves are presented in Figure 3. The accelerations of oscillating motion $X''(t)$ – a base for the dynamic properties evaluation of the selected transition curves, are presented in Figure 4. The accelerations of the oscillating motion *X''(t)* have been computed using formula (13) with the assumed values of the parameters (for a rail vehicle): free oscillation frequency ω = 3.5 1/s, Lehr's damping coefficient D = 0.175, the same constant velocity v = 120 km/h.

In order to express the evaluation results in terms of real numbers (i.e. dynamic criterion value) and to extract oscillations, the signal of the acceleration of oscillating motion *X''(t)* is filtered using a IIR highpass Butterworth digital filter of 11th order [8] with normalized frequencies 0.025 or 0.055, depending on the assumed transition curve length. The filtered signals (with normalized frequency 0.025) of the acceleration of oscillating motion *X''(t)* for the evaluated transition curves are presented in Figure 5. The filter parameters have been adjusted using `iir` and `flts` functions from SciLab v. 5.4.0.

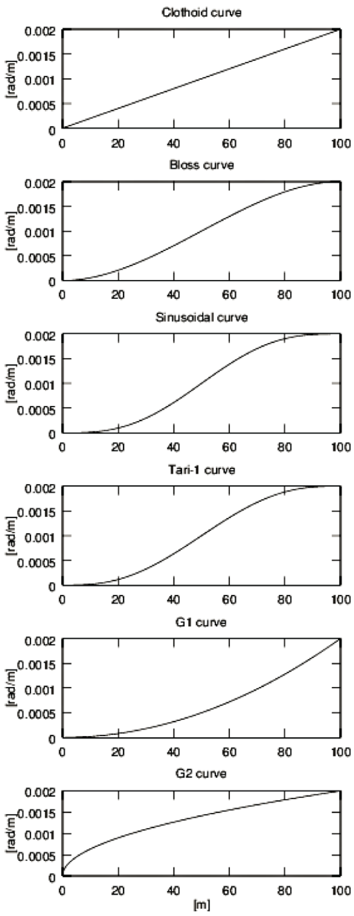

Fig. 3. Curvatures of the evaluated transition curves (l=100 m)

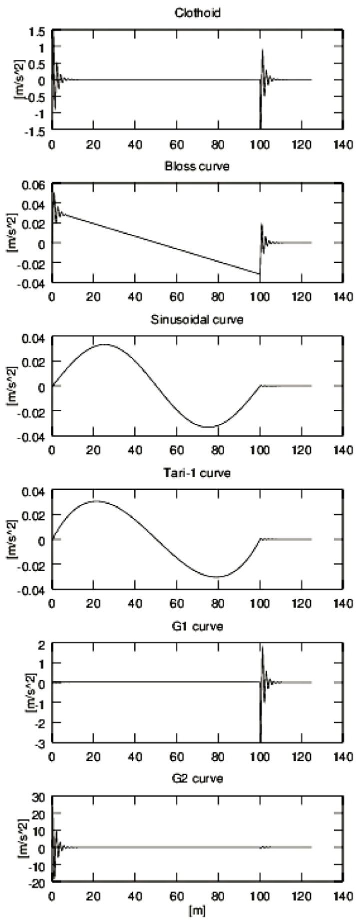

Fig. 4. Acceleration in oscillation motion *X''(t)*

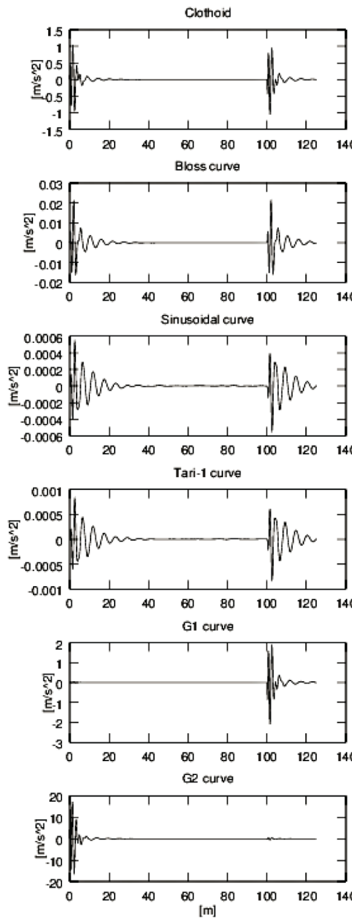

Fig. 5. Filtered acceleration in oscillation motion $X''(t)_F$

The two areas of the increased dynamic interactions – the transition curve origin and end points are noticeable in Figures 4 and 5. Usefulness of the filter application (Figure 5) is particularly evident in comparison of Bloss curve, sinusoidal curve and Tari-1 curve. The analysis of the filtered acceleration in the oscillating motion *X''(t)* indicates significantly better dynamic properties of a sinusoidal curve and a Tari-1 curve (lower maximal acceleration amplitudes in the neuralgic areas – in Figure 4 practically imperceptible) in comparison to a Bloss curve. There is a tight relationship between the dynamic properties of the transition curve and a continuity class of its curvature function [11, 12, 13, 14]: higher class of continuity, especially continuity at joining points not internal continuity along transition curve, leads to lower dynamic interactions in the vehicle-route system.

Much less favourable dynamic properties has been found for clothoid curve, G1 and G2 curves. In Figures 6-7 accelerations in oscillating motion *X''(t)* at the curves origin points (joining of straight line and transition curve) and curves end points (joining of transition curve and circular arc) have been presented. G1 curve has advantageous dynamic properties near its origin point, having by far the worst properties in the end area. G2 curve conversely: favourable properties in the end area (better than clothoid curve) are occupied by significantly poorer properties in the origin area.

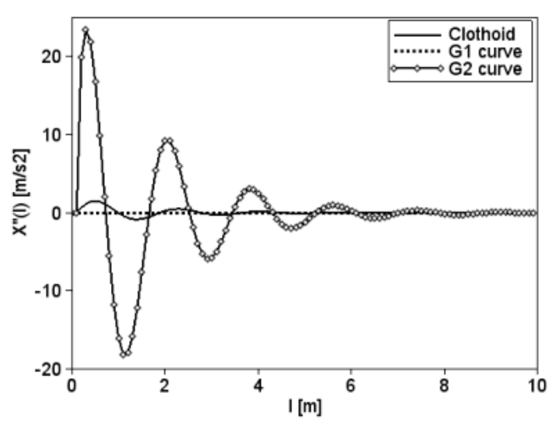

Fig. 6. Acceleration in oscillation motion *X''(t)* in the origin area: clothoid, G1 and G2 curves

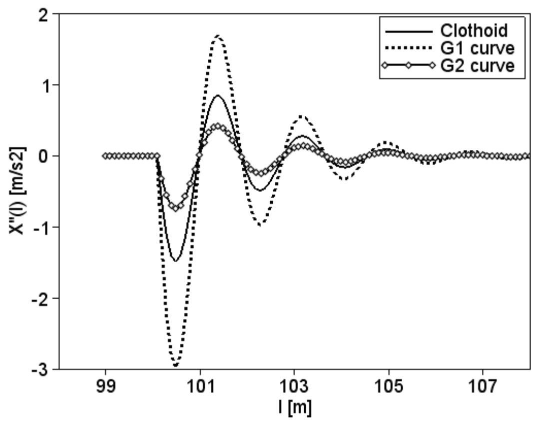

Fig. 7. Acceleration in oscillation motion *X''(t)* in the end area: clothoid, G1 and G2 curves

The conclusions resulting from above analysis were reflected in dynamic criterion values, presented in Table 2.

Table 2. Dynamic criterion values for selected transition curves

Curve type	Dynamic criterion value	Curve type	Dynamic criterion value
Clothoid	52,1561	Tari-1 curve [12]	0,0507
Bloss curve	1,1580	G1 curve [13]	52,2726
Sinusoidal curve	0,0335	G2 curve [13]	438,5470

The obtained dynamic criterion values indicate a significant advantage of sinusoidal curve and Tari-1 curve over clothoid, G1 and G2 curves – a conclusion is consistent with the results of the analysis based on acceleration of oscillating motion *X''(t)* diagrams.

Based on the data from Table 2, it is expected that in the optimization process with dynamic criterion transition curves with non-linear, symmetrical curvature (criterion equally considered the dynamic interactions occurring in the origin and end areas of the curve) will be obtained.

Apart from the dynamic criterion, fitness function (14) includes an element connected with minimization the distance between center C2 of a given circular arc and center C of the obtained arc with radius perpendicular to transition curve tangent at the end point (Figure 1).

$$FF = w_d \int_0^{l+\delta} X''(t)_F + w_p \|C - C_2\| \to min \qquad (14)$$

where:

w_d, w_t and w_b – are arbitrarily adjusted weights,

δ – length of the straight line section on which dynamic effects occur after leaving a transition curve with length *l*.

4.2 Optimization algorithm

Particle Swarm Optimization (PSO) algorithm [9] has been applied for determining curvature. PSO performs a population-based search using moving particles to represent potential solution – searched curvature, defined unambiguously by control points coordinates $\{\boldsymbol{P_i}\}_{i=0}^{3}$ (Table 3).

Table 3. Structure of a particle

x-coordinates				y-coordinates			
$P_{0x}=0$	P_{1x}	P_{2x}	P_{3x}	$P_{0y}=k_1$	P_{1y}	P_{2y}	$P_{3y}=k_2$

Among 8 coordinates of control points $\{\boldsymbol{P_i}\}_{i=0}^{3}$, 3 have assumed values (cells with gray background in Table 3) while five others are obtained in the optimization process carried out using PSO toolbox v. 0.7-1 [16] in SciLab v. 5.4.0. Assumed values guarantees a priori meeting conditions (2) and (3) imposed on the transition curve curvature [10].

In PSO algorithm each particle is characterized by its fitness value, current position, current velocity and a record of its past performance. Particle change their positions and velocities in the direction dependent on the best recorded own position, position of other particles and current velocity. Velocity of each particle in the swarm is updated in each iteration according to the following formula:

$$v_{id} = \omega \times v_{id} + C_1 \times Rand(\,) \times \left(p_{id}^{best} - p_{id}\right) + C_2 \times Rand(\,) \times \left(p_{gd}^{best} - p_{id}\right) \qquad (15)$$

where: ω – inertia coefficient, changing during the execution of the algorithm as follows:

$$\omega = \omega_{max} - n \times \frac{\omega_{max} - \omega_{min}}{N} \qquad (16)$$

N – total number of iterations,

n – current iteration number,

C_1, C_2 – individual and swarm learning coefficients,

Rand() – random value from interval (0,1);

p_{id}^{best} – the best recorded so far particle position (individual best position);

p_{gd}^{best} – the best recorded so far position of other particles (global best position).

Position of each particle is updated in each iteration according to the equation:

$$p_{id} = p_{id} + v_{id} \tag{17}$$

PSO flow chart is presented in Figure 8.

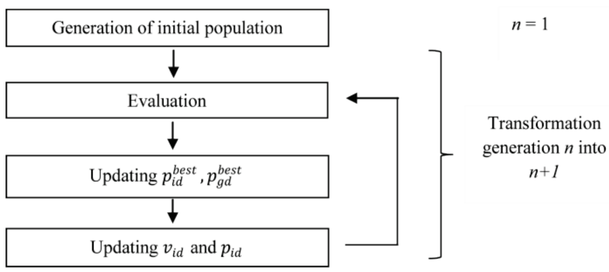

Fig. 8. Particle Swarm Optimization algorithm flow chart

5. RESULTS

The presented method of curvature modelling has been applied to determine a transition curve joining two circular arcs Ω_1 and Ω_2 with consistent curvatures (Table 4). The arc Ω_2 is inside the arc Ω_1; location of the centre C_2 of the arc Ω_2 has been obtained as a result of K_0 curve (6) construction using the universal method of curvature modelling (Section 2).

Table 4. Geometrical parameters of two circular arcs with consistent curvatures

Location of the arc center		Radius R [m] of the arc	
C_1	(0;700)	R_1	700
C_2	(41,22; 506,42)	R_2	500

Due to application of the element $w_p \| C - C2 \to min$ in the fitness function (14) the presented method enables to obtain transition curves with a tangent perpendicular to the radius of a given circular arc Ω_2 at the end point – a given location of the arc centre C_2 is preserved and treated as a geometrical constraint that should be satisfied. This feature is valuable: through adjusting the weight w_p in the optimization process it is possible to obtain a curvature of the transition curve preserving a given centre location of the arc Ω_2.

The control points of curvatures presented in Table 5 have been obtained as a result of two optimization processes lasted n=200 iterations each using the fitness function (14) with wages w_d and w_p. It has been assumed that $P_{0x}=0$, $P_{0y}=1/R_1$, $P_{3y}=1/R_2$.

Table 5. Coordinates of the control points of resulting curvatures

No	w_d	w_p	P_{1x}	P_{2x}	P_{3x}	P_{1y}	P_{2y}
1	1	2	91,76	127,16	392,03	0,001428892	0,001999588
2	1	0,1	143,21008	143,2101	301,03	0,001428572	0,001999805

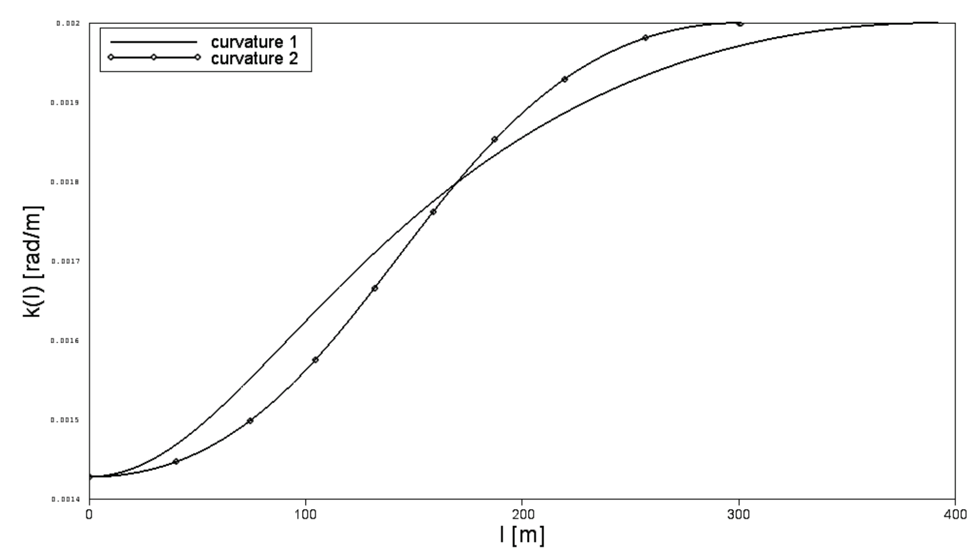

Fig. 9. Curvatures obtained in the optimization processes

Table 6. Parameters of the resulting transition curves

No	w_d	w_p	Dynamic criterion value	Shift of the arc Ω_2 centre $\|C - C_2\|$ [m]	Joining point of transition curve and arc Ω_2 [m]	Transition curve tangential angle at the end point	Length [m]
1	1	2	1,57047	0,0001965	(363,29; 123,96)	0,6998911	392,06
2	1	0,1	0,5904005	1,0011463	(289,04; 71,05)	0,51801	301,11

On the basis of obtained curvatures (Figure 9), using formulas (7)-(9) coordinates of the resulting transition curves are determined (Figure 10). The geometrical parameters of the resulting transition curves are presented in Table 6.

Both optimization processes have been carried out using parameters presented in Table 7. Parameters ω_{max} , ω_{min} , C_1 , C_2 and N are used in formulas (15)-(16).

Table 7. Parameters of the PSO algorithm

Parameter	Value
Initial inertia parameter ω_{max}	0,9
Final inertia parameter ω_{min}	0,4
Individual learning coefficient C_1	0,7
Swarm learning coefficient C_2	1,47
Iterations number N	200

The course of the optimization processes is presented in Figure 11. Both processes converge to optimal solution after approximately 100 iterations.

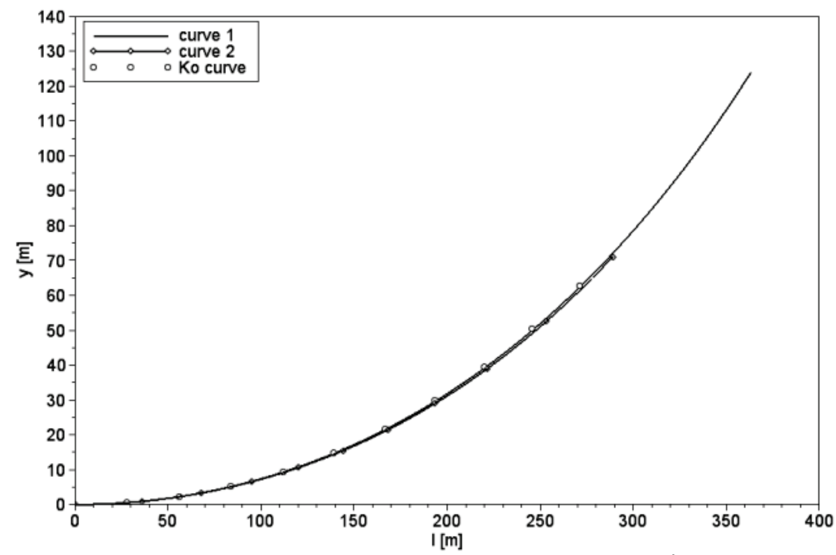

Fig. 10. Resulting transition curves compared with K^0 curve

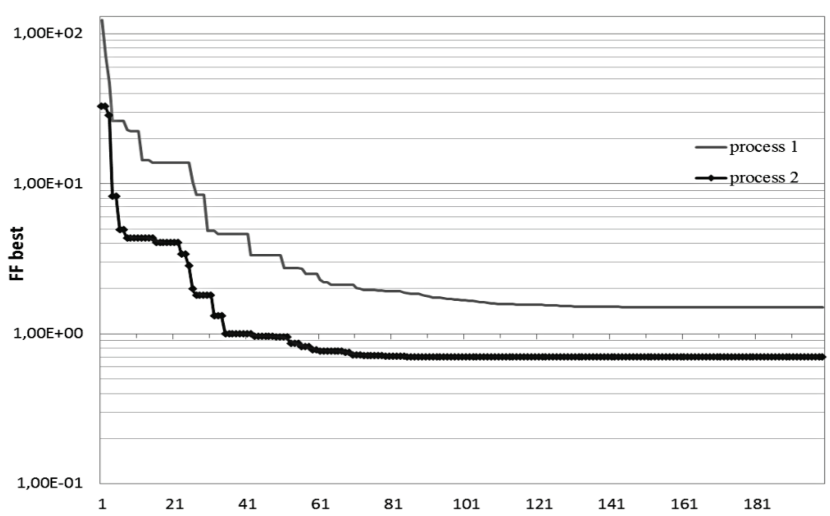

Fig. 11. Fitness function value in successive iterations of optimization processes 1 and 2

The transition curve 1, obtained as a result of the process 1, in which greater emphasis was put on the minimization of required shift of the centre of the arc Ω_2, based on dynamic criterion value (Table 6) has less favourable dynamic properties than transition curve 2 (1,57047 to 0,5904). The main objective of the second optimization process, expressed by weight ratio w_d/w_p, was minimization of the dynamic interactions occurring in the vehicle-route system. The dynamic criterion values results from the acceleration in oscillating motion *X''(t)* (Figure 12) or precisely on its filtered version $X''(t)_F$ (Figure 13). Both obtained transition curves have more advantageous dynamic properties than K^0 curve (the dynamic criterion value for K^0 curve equals 52,16).

normalized frequency 0.055. The filtered signal $X''(t)_F$ is integrated during computation of the dynamic criterion values – formula (14). The most advantageous dynamic properties of transition curve 2 result from the lowest maximal amplitude of filtered signal $X''(t)_F$. ($\approx$0,0012 m/s^2). The maximal amplitude of the filtered signal for K0 curve is approximately 10^2 times higher.

6. CONCLUSIONS

The method of curvature modelling, presented in this paper, enables flexible shaping of curvature of transition curve. Obtained transition curves have advantageous dynamic properties and satisfy given geometrical constraints: preserve location of the

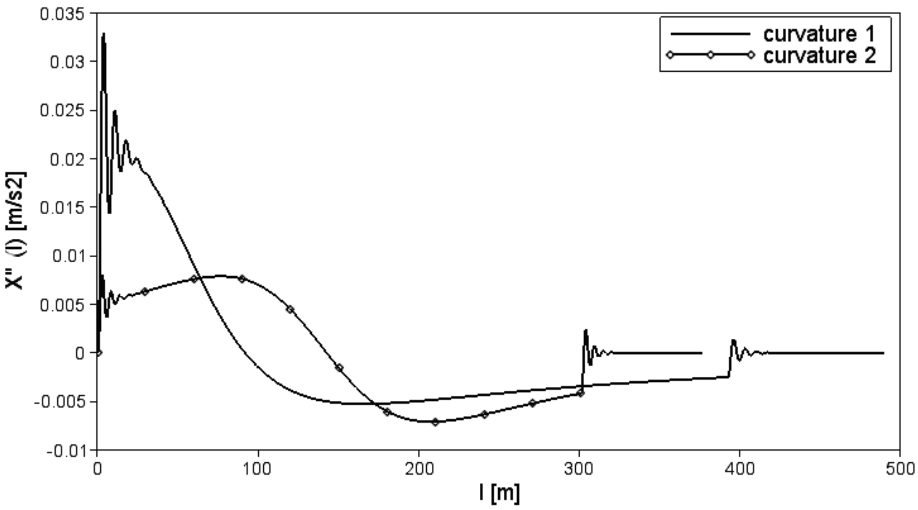

Fig. 12. Acceleration in oscillating motion *X''(t)*

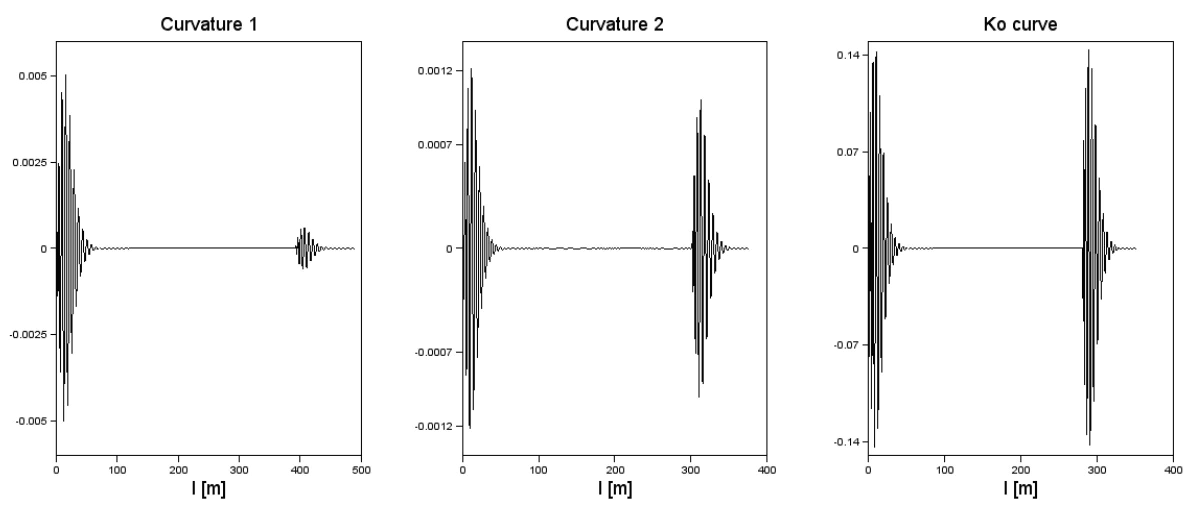

Fig. 13. Filtered acceleration in oscillating motion $X''(t)_F$

centre of the arc Ω_2, preserve conformity of tangential angles at joining points.

In Figure 13 the filtered acceleration of oscillating motion *X''(t)* is presented. The filtration process is carried out using a IIR highpass Butterworth digital filter of 11th order [8] with

The essential novelty of the method is to describe curvature of the designed layout as a cubic C-Bezier curve [19] and shape it in the optimization process with the dynamic criterion by changing locations of the control points. The experiments proved that Particle Swarm Optimization algorithm can be efficiently applied to search for optimal coordinates of control points that determine unambiguously cubic C-Bezier curve.

REFERENCES

[1] Baykal, O. : On concept of lateral change of acceleration, Journal of Surveying Engineering, 122(3) 1996, 132-141.

[2] Baykal, O, Tarı E, Coşkun Z, Şahin M, A New Transition Curve Joining Two Straight Lines, Journal of Transportation Engineering, ASCE, 123(5),337-345, 1997.

[3] Cai H., Wang G.: A new method in highway route design: joining circular arcs by a single C-Bezier curve with shape parameter. Journal of Zhejiang University Science A 2009, 10(4).

[4] Dahlquist, G., Björck, A. Numerical methods, Prentice-Hall 1974.

[5] Delint I, Siti M., Aida A.: Particle Swarm Optimization for NURBS curve fitting. Proceedings of the 2009 Sixth International Conference on Computer Graphics, Imaging and Visualization, pp. 259-263. DOI 10.1109/CGIV.2009.64

[6] Habib Z., Sakai M.: G^2 Pythagorean hodograph quintic transition between two circles with shape control. Computer Aided Geometric Design 2007, nr 24.

[7] Habib Z., Sakai M.: On PH quantic spirals joining two circles with one circle inside the other. Computer-Aided Design 2007, nr 39.

[8] Ifeachor, E., Jervis, B. : Digital Signal Processing. A Practical Approach, Addison-Wesley, 1998.

[9] Kennedy J., Eberhart R.: Swarm intelligence, Morgan Kaufmann Publishers, San Francisco, 2001.

[10] Koc W.: Elements of the theory of track design (in Polish), Gdansk University of Technology Publishing, Gdansk 2004.

[11] Koc, W., Mieloszyk, E.: Comparative analysis of selected transition curves using dynamic model, Archives of Civil Engineering, 33(2) 1987, 239-261.

[12] Koc W., Palikowska K.: Dynamic properties evaluation of the selected methods of joining route segments with different curvature (in Polish), XVI International Scientific Conference "Science, Industry and Transport Computer Support Systems − TransComp", Zakopane 2012.

[13] Koc W., Palikowska K.: Analysis of the curvature modeling methods –Bezier curves against the analytical method (in Polish). Scientific-Technical Papers SITK, 3(99), s. 225-237, Cracow 2012.

[14] Mieloszyk, E., Koc, W.: General dynamic method for determining transition curve equations, *Rail International-Schienen der Welt*, 10 1991, 32-40.

[15] Palikowska K.: Design of railway track geometrical layout using evolution programming, PhD Thesis (in Polish), Gdansk University of Technology, 2002.

[16] Salmon S., Baudin M., Cornet A.: Particle Swarm Optimization Toolbox, 2012, http://forge.scilab.org/index.php/p/pso-toolbox/

[17] Tari, E.: The new generation transition curves. ARI 54(1), Istanbul Technical University 2003.

[18] Tasci, L., Kuloglu N.: Investigation of a new transition curve. The Baltic Journal of Road and Bridge Engineering 6(1) 2001÷223 DOI: 10.3846/bjrbe.2011.04

[19] Zhang J.: C-curves: an extension of cubic curves. Computer Aided Geometric Design 13(3) 1996:199-217, DOI: 10.1016/0167-8396(95)00022-4.

Katarzyna Małgorzata Palikowska
Gdansk University of Technology, Poland
katpalik@pg.gda.pl