Cooperative Games in Integration of Supply Chain on the Example of Purchasing Groups

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Purchasing group can improve the chance of a higher return for individual entities in the supply chain. The greater sales occurs between companies, the higher the probability of deepening integration between them. Functioning of the purchasing group is, however, dependent on the justice of imputation the profits generated by it (discounts, cost reduction) for its entities. Article presents the use of cooperative game, in order to explain the possibility of optimizing and sharing the profits (payoffs), and in a longer perspective, build an integrated supply chain.

Keywords: lot sizing, purchasing groups, game theory, ABC method.

1. INTRODUCTION

Integration of supply network members is one of the basic premises of whole networks competing on the market, which is why so much attention is paid to the tools and methods supporting this concept. One method of designing an integrated supply chain is the ABC method, described in this article. It justifies the establishment of links between companies according to rule "equal to equal". In business practice that situation shall not always occurs. The market power of companies is one of the sources of power of a supply chain leader. And the power can lead to the desire of abusing it. In this case, equal opportunity for other chain actors can take place by concatenating their forces, for example in the form of purchasing group (or sales group).

The problem, however, remains – how to justly distribute the profits of a purchasing group. The benefits here are not only earned discounts. Joint purchases bring with them cost savings, which are described later in this article. Condition of a good co-operation is therefore equitable distribution of profits. Here comes the game theory, that widely discussed problems of the so-called payments split known as imputation of payoffs. The cases related to games with imputation of payments in purchasing groups are called ELS - Economic Lot Sizing Games.

2. SPECIFYING THE DEGREE OF INTEGRATION BY ABC METHOD

The degree of integration is determined in different ways, one of these is the ABC method. It is based on the demonstration of volume of revenues of both suppliers and customers in a given period, such as the last twelve months, and assign the volume of revenues between the various elements based on Pareto method. The scale of revenues should be determined by two letters, the first of which relates to the supplier and the second to recipient. For the supplier, the letter A indicates that the recipient is in the top 20% of the largest which generate 80% of revenues, B in the group a further 30% of the customers which generate 15% of revenue and C in a group of 50% of the customers which generate only 5% of revenue. The same principle will apply in determining the recipient, which will concern 20, 30 and 50% of providers that generate accordingly 80, 15 and 5% of the material costs. While it is easy to classify suppliers and customers into groups A, B or C, it may be much more difficult to indicate which group turnover of the company belongs to, in relation to turnover of suppliers and customers. However, it is necessary to be aware of how company is important to their suppliers and customers from the perspective of realized revenues, which is the source of generating profit.

Thus, the volume of mutual trade and its reference to the revenues of each of the parties, plays an extremely important role in building relationships and is the lever to take any non-standard forms of cooperation. Thus the relation AA certainly will be the foundation for the integration of the supply chain[1]. Supply network with selected items of ABC method is presented in Figure 1.

Very important from the point of view of maintaining a strong competitive position is to ensure an appropriate level of cost of acquisition of materials and services. Lack of alternative sources of obtaining materials or services results in a stronger position of the supplier whose aim is, after all, to obtain the highest profit possible. It must therefore be assumed that these dependencies need to be affected by a higher level of costs. There is also the risk of dependence on customers and thus a situation in which the recipient has a much stronger position and may dictate adverse conditions for cooperation degrade the profitability of the business[2].

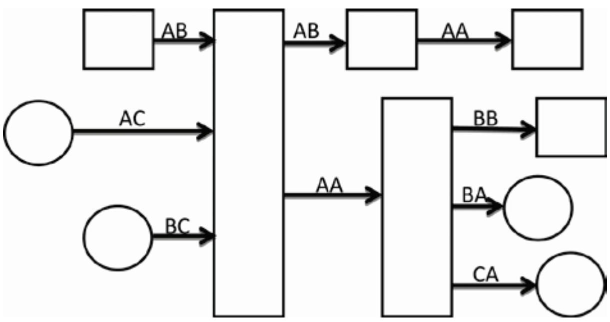

Fig. 1. Determination of integration degree in the supply chain with use of the ABC method
Source: Urbańczyk T., *Rozwój sieci dostaw poprzez wdrażanie koncepcji łańcucha solidarności*, PhD Thesis, Poznań 2012, p. 96.

AA relationships are the foundation for building a mutual integration of the supply chain. Therefore, efforts should be made to establish such a relationship. Relationships with suppliers BA, CA, AB, BB, CB, AC, BC, CC should aim to select comprehensive suppliers to achieve integration AA. This means that the organization should strive to consolidate their purchases from a single supplier in a given product range, and avoid an ad hoc purchases from multiple vendors[3].

3. COOPERATION GAMES

In game theory, the games which are called cooperation games are those in which all equilibrium of pure Nash strategy are situations in which players choose the same or corresponding strategies. In other words, if the participants in a game can make binding commitments to coordinate their strategies, then the game is cooperative. The solution with coordinated strategies is a cooperative solution. Coalition is a group of players who coordinate their strategies.

An example of the coordination game is a game in the direction of traffic. Both the solution to "all drive on the left side" and "all drive on the right side" is the Nash equilibrium, but none is better than another. However, in the real situation, only one of these strategies is finally chosen, by matching the (reconciliation, cooperation, coordination) between the players.

The main issue will be the imputation of payments (value) between all members of the coalition. This imputation will be identified with the solution of the game. There are numbers of solution concepts for cooperative games. Two of them are important for this paper: the core and the Shapley value, where it must be considered as superadditive games, means such where the value of the sum of two disjoint coalition is not less than the sum of their values - the joint coalition is profitable.

The core of a cooperative game consists of all imputations (if there are any) that are stable in the sense that there is no individual or group that can improve their payoffs (including side payments) by dropping out or reorganizing to form a new or separate coalition. Side payments occur when a part of the payoff is transferred from one member of a coalition to another, so that no member of the coalition needs to be worse off as a result of adopting the coordinated strategy of the coalition. Side payments will always be possible in a game with transferable utility (correlated with money payoffs) such as purchasing group problem[4].

The Shapley value is also applied for supper additive games in coalition function form, but within those limits it has the advantage of

[1] Urbańczyk T., Rozwój sieci dostaw poprzez wdrażanie koncepcji łańcucha solidarności, PhD Thesis, Poznań 2012, pp. 95-96.
[2] Ibidem, p. 96.
[3] Ibidem, p. 97.
[4] R.A. McCain, Game Theory – A Nontechnical Introduction to the Analysis of Strategy, World Scientific, p. 403-415.

uniqueness: there is always exactly one Shapley value. This value is based on a concept of a marginal contribution[5]. Shapley presented the value as an operator that assigns an expected marginal contribution to each player in the game with respect to a uniform distribution over the set of all permutations on the set of players. Specifically, let π be a permutation (or an order) on the set of players, in example, a one-to-one function from N onto N, and let us imagine the players appearing one by one to collect their payoff according to the order π. For each player i we can denote by $p=\{j:\pi(i)>\pi(j)\}$ the set of players preceding player i in the order π. The marginal contribution of player i with respect to that order π is $v(p_\pi^i \cup i)-v(p_\pi^i)$. If permutations are randomly chosen from the set π of all permutations, with equal probability for each one of the n! permutations, then the average marginal contribution of player i in the game v is:

$$\phi_i(v)=\frac{1}{n!}\sum_{\pi\in\Pi}\left[v(p_\pi^i \cup i)-v(p_\pi^i)\right],$$

which is the Shapley definition of value[6].

Cooperative game theory focuses on what groups of players can achieve rather than what individuals can do. As a result, it typically assumes complete information. On the other hand, mechanism design in non-cooperative game theory aims to design a game that achieves the desired social outcome even when agents are self-interested, strategic and have private information. Sometimes, the boundary between non-cooperative game theory and cooperative game theory is not always clearly cut (because non-cooperative game theory is sometimes also involved in cooperating)[7].

4. PURCHASING GROUPS IN COOPERATIVE LOT SIZING GAMES

The functioning of the purchasing group based on the assumption of "a big can do more" is reasonable economies of scale of implementation several smaller contracts, which is the very purpose of its establishment. The use of purchasing group power (large one-time or recurring contract) is the basis for minimizing the costs associated with the distribution, especially with realization of one transportation orders set[8].

Consider a set of retailers N ={1;…;n} that sell the same item. They all have a known demand for a t-period model horizon. All retailers buy the items at the same manufacturer. When a retailer places an order, the manufacturer charges ordering cost and production cost, which is linear in the amount of items ordered. Furthermore, when a retailer carries inventory from one period to the next period to satisfy future demand, holding costs are incurred. We assume that in a single period holding costs are equal for each retailer. Now a single retailer tries to minimize its total ordering, production and holding costs. Note that this is exactly the situation as in the Economic Lot Sizing (ELS) problem, where setup cost in the ELS problem corresponds to ordering cost in the ELS game. If a collection of retailers cooperates in the above setting, a cost saving may be obtained. Namely, instead of placing individual orders the retailers can place a joint order and save ordering costs[9].

We search for the ordering quantity q_t in a way that total cost are minimized. The total costs consist of holding costs c_t^h per stored item, fixed ordering costs c_t^s (incurs whenever an order is placed in period t) and unit ordering costs c_t^p. Depending on the order quantities and the demand, there are I_t units of the item on stock at the end of period t. Formally, each player $i\in N$ has a demand d_{it} that should be met in period t. If all players of set N cooperate they face a join demand. An order in period t is placed when $x_t=1$, 0 otherwise. The problem to be solved can be couched as[10]:

[5] Ibidem, pp. 418-421.

[6] E. Winter, The Shapley value. In: Aumann RJ, Hart S (eds.) Handbook of Game Theory with Economic Applications, vol. 3/2002, Elsevier, chap 53, p. 2028.

[7] L. Boongasame, Survey on Buyer Coalition Mechanisms, Applied Mathematical Sciences, Vol. 7/2013, No 5, p. 232.

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[9] W. Heuvel, P. Borm, H. Hamers, Economic Lot-Sizing Games (17 2004, 11), p. 6. ERIM Report Series Reference No. ERS-2004-088-LIS. Available at http://ssrn.com/abstract=636804.

[10] J. Drechsel, Cooperative Lot Sizing Games in Supply Chains, Springer-Verlag, Berlin Heidelberg 2010, p. 63-65.

$$c(N) = \min \sum_{t=1}^{T} (c_t^p q_t + c_t^s x_t + c_t^h I_t)$$

$$d_t(N) = \sum_{i \in N} d_{it} \qquad t = 1,....,T$$

$$M = \sum_{t=1}^{T} d_t \qquad t = 1,....,T$$

$$I_t = I_{t-1} + q_t - d_t(N) \qquad t = 1,....,T$$

$$q_t \le M \quad \mathrm{x_t} \qquad t = 1,....,T$$

$$I_t, q_t \ge 0 \qquad t = 1,....,T$$

$$x_t \in \{0,1\} \qquad t = 1,....,T$$

$c(N)$ represents the total costs for ordering jointly in the grand coalition N. By construction, total costs for an arbitrary coalition S are positive ($c(S) \ge 0$ for all $S \subseteq N$) and total costs for an empty coalition equal 0 ($c(0) = 0$)[11].

The question arises how to allocate the total costs to the single members of the coalition. From a theoretical perspective, allocation rules are often tested based on the number of attractive properties they satisfy from a fairly standard list of requirements. This list includes several reasonable properties such as symmetry, efficiency, additivity, individual rationality, etc. Of the many popular allocation rules from theory, such as the Shapley value, the nucleolus and the compromise value, the Shapley value turns out to be the most attractive for the savings game. Moreover, recall that we assumed p(q) is a convex function and qp(q) is concave and increasing in q. This seems to be a reasonable assumption that holds for most schedules in practice. In particular, the total payments made to the seller, as one would expect, increases as the quantity purchased becomes larger, and this increase sees a diminishing return. When this property is factored in, the savings game turns out to be convex. Convexity implies the super-additivity of the game; that is, when any two disjunctive coalitions join, the total savings generated by their members increase. A direct implication of this fact is that the Shapley value satisfies a myopic stability property (in example the Shapley value belongs to the core when the game is convex, and no subset of players wants to defect from N). The Shapley value with stands commonly used tests of fairness and equity. Moreover, common problems associated with purchasing groups, such as monotonicity of payoff with respect to contribution and weak free rider issues, are minimal when the Shapley value is used. Further, the nucleolus and compromise value do not yield the grand coalition in the farsighted sense, are less intuitive, and therefore harder to implement in practice. The Shapley value also has the advantage that it has robust approximations, which is convenient for practical applications[12].

5. APPLICATIONS

Game theory has become a common tool in defining the relationship between firms in the supply chain. Presented in this article the issue of optimizing and sharing of benefits arising from the functioning the purchasing group in the supply chain is one of many possibilities that it carries. Concepts of searching for the optimal solution in the core game and the profit-sharing from Shapley value will apply in simple situations, where access to information is good (in theory unlimited) and market situation uncomplicated. For more complex situations an extended operation algorithms such as Multi-level Lot Sizing[13] or an Inverse Economic Lot Sizing Approach[14] would be beneficial.

If purchasing group finds the imputation as fair, relationships in the supply chain will foster further trade and power of the leader will be less. In other words, the chain is closer to the relational situations AA, which will follow with the construction of a chain of solidarity.

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