The Model of Reverse Logistics, Based on Reliability Theory, with Elements' Rejuvenation

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Reverse logistics understood as the process of managing reverse flow of materials, in-process inventory, finished goods and related information has become one of the logisticians’ key areas of interest. Nowadays a growing number of companies realize the meaning of that field of logistics. Reuse of products can bring direct advantages because company uses recovered components instead of expensive new ones. On the base of literature overview the aims of the paper were established. It proposes the model that uses the reliability theory to optimize recovery policy parameters. The criterion of the policy parameters optimization is the total cost of the production, recovery and services during warranty period. Presented model is a development of existing approaches because it widely analyses the aspect of returns condition in the recovery policy optimization.

**Keywords:** reverse logistics, production, returns, repair options, reuse, reliability theory.

1. INTRODUCTION

Reverse logistics understood as the process of managing reverse flow of materials, in-process inventory, finished goods and related information has become one of the logisticians’ key areas of interest. Nowadays a growing number of companies realize the meaning of that field of logistics. Reuse of products can bring direct advantages because company uses recovered components instead of expensive new ones.

Material flow in the supply chain (reverse logistics) can be represented by a Sankey diagram (Figure 1), in which the individual arrows indicate the type of returns, direction of flow and the volume / flow rate of returns. Stream size is proportional to the width of the individual blocks. Based on the presented definition of reverse logistics and studied literature [1, 2, 5, 14, 18, 19] it is possible to define the reverse logistics area of interest, which includes the following groups:

- returns of raw materials, intermediates and final products from the production phase (surplus raw materials, defective products rejected by quality control, by-products and waste generated in the production process);
- product returns and packaging from the phase of distribution (products damaged during transport, reusable packaging, excess inventory, unsold products);
- decommissioned objects (products returned because of changing the purchasing decision or a qualitative defect, parts replaced during the maintenance process, products that fail during warranty period which repair was impossible and other and waste).

![Figure 1. Material flow in reverse logistics](image-url)
In this paper we take into consideration the third stream of returns – products returned because of one the components failure. Failed components are thrown away and good ones can be used again after performing the recovery. Types of recovery operations that can be performed can be defined on the basis of the classification presented in [19]. On the basis of these publication one can distinguish the following types of recovery:

- direct reuse/resale,
- repair,
- refurbish,
- remanufacturing,
- cannibalization,
- recycling.

More detailed descriptions of different types of recovery are presented in [5, 6, 10, 14, 18, 19, 20]. Cannibalism as a method of maintenance policy is discussed in [3,4, 7, 8, 11, 12, 15, 16, 17].

Because the components considered in our paper as reusable aren’t damaged and failed parts are replaced with new ones, we take into consideration only: direct reuse of old components, refurbishment and remanufacturing of the products which increase the reliability of the old components. According to the literature remanufacturing brings products back to as good as new condition but it is possible to imagine that even in remanufacturing there can be the difference in the achieved level of components reliability. It strongly depends on used methods or technology. In this paper we assume that better condition of the component can be achieved after incurring higher costs.

Literature review that have been done so far allowed to define the main shortages of existing reverse logistics models:

- most of models assume single-component product,
- they are usually based on the assumption that recovered products are as good as new ones,
- very few articles optimize the threshold age of returns, but none of them gives any guidelines what kind of recovery technology or method should be applied.

On the base of literature overview the aims of the paper were established. It proposes the model that use reliability theory to optimize recovery policy parameters for the case when:

- a new, multi-component object is produced from the mix of new and reused components,
- reused elements are not good as new,
- depending on the age of returns and its reliability characteristics, various actions may be undertaken:
  - a direct reusing of the component,
  - an element repair, which allows to improve its condition with additional cost,
  - disposal.

The criterion of the policy parameters optimization is the total cost of a production, recovery and services during warranty period. In the next sections the main assumptions and the cost model of the reusing policy are presented. On that base the sensitivity analysis of the model is presented and the paper ends in short conclusions.

2. MODEL OF THE REUSING POLICY

The model that is presented in the paper is based on the following assumptions:

- A company produces an object composed of two elements (A and B). The product has a series reliability structure, so it fails after any components failure.
- A failure of each element occurs independently on other components' faults.
- If the product fails during the warranty period, it is returned to the manufacturer and he has to pay some penalty cost (e.g. the cost of a new product).
- The component B of the product may be reused in a new production, if it was not the cause of the product failure and its reusing is profitable for the producer. All A components are new in a new production.
- The products are returned as soon as their lives are ended and then B components may be:
  - stored in a stock until new production batch running,
  - recovered in order to improve their condition with additional cost,
  - disposed.
- Demand for the products is determined and fixed.
• Production batch are run periodically in established moments and new products are sold at once.

Despite the fact that companies realize the potential of reusing products, the question “is it worth to do it”, is not so simple to answer. Within main reasons for products reusing are: difficulties with raw material supplying, high cost of utilization of returned and damaged products or lower cost of reusing of products' components than buying new ones. The objective of the presented model is to estimate profitability of using returned and recovered elements in a new production, in the case when they are not as good as new.

According to the assumptions, before every production beginning, the manufacturer has to make the decision: which of returned elements B should be used in the new production. The usage of recovered components decreases production costs but also increases the risk that additional costs occur because of larger amount of returns during the warranty period.

Very few models presented in literature (e.g. [9,13]) analyze the threshold age of returned components for which direct reusing in the new production is profitable. The considered cost model has taken into account the total costs of: new and reused components acquirement and services during warranty period of the sold object. The model proposed in this paper is the development of the previous ones. It takes into consideration alternative action – element’s rejuvenating with additional cost. The idea is presented in Fig. 2.

The object, that was sold in the $T_P$ moment, may be returned to the producer during warranty period $T_W$. If the return was caused by an element A failure, the B component is returned in the age of $T$ and it may be reused in the next $T_P$ moment. Thanks to some recovery activities with additional cost, the age of elements may be reduced by $\Delta T$ and its residual reliability may be increased with $\Delta R(t)$ value. The potential costs and profits of this solution may be estimated by the following formula:

![Figure 2. The concept of returned element recovery before use in a new production batch](image-url)
The total profit of a possible solution (Eq. 1) depends on the savings resulting from the reusing of “old” elements instead of new ones which must be bought at a price $C_B$. On the other hand potential profits are lowered by the costs of component rejuvenating ($C_R$) that is dependent on the length of $\Delta T$, the cost of element’s age identification (e.g. by individual number of the component) and the cost of a potential growth of failure probability of the object during the whole warranty period. The profit depends upon two variables of time being in various configurations (Fig. 3,4):

- the reduced age of the component after all taken recovery activities ($T - \Delta T$), which determines its costs coming from reliability decrease ($C_1$ in Fig. 3,4),
- the age of return ($T$) and the length of period $\Delta T$ that determine the costs of rejuvenating ($C_2$ in Fig. 3,4), depending on the form of the function $C_R$.

\[
P(T, \Delta T) = C_B - C_R(\Delta T) - C_M - [F_1(T - \Delta T + T_w) - F_2(T_w)] - C_O \quad (1)
\]

\[
F_1(T - \Delta T + T_w) = 1 - \frac{R_B(T - \Delta T + T_w)R_A(T_w)}{R_B(T - \Delta T)} \quad (2)
\]

\[
F_2(T_w) = 1 - R_B(T_w)R_A(T_w) \quad (3)
\]
As one can see in Fig. 3,4 the cost resulting from a potential unreliability growth is constant for a given relation: \( T - \Delta T \) and the costs of elements rejuvenating is constant for a given \( \Delta T \) (for cost function dependent on \( \Delta T \)). The total profitability of the elements’ reusing is the effect of the sum of costs \( C_1 \) and \( C_2 \) in comparison to the cost of a new component.

The models given in the literature [9,13] allow to determine the threshold age of return for which it is still profitable to reuse a component that is not \textit{as good as new} in a new production. However, if there is a practical possibility to execute some recovery activities, the threshold age seems to be impossible to determine, because a “new” object containing even “very old”, but rejuvenated element may be finally cheaper than the one comprised of all new elements. Thus, the most economical decision should be taken on the basis of the expression given in Eq. 1 and the element reusing is economically grounded if total profit \( P(T, \Delta T) \) is a positive number.

3. SENSITIVITY ANALYSIS

In order to assess the influence of chosen parameters upon outputs of the presented model, sensitivity analysis have been conducted. The aim of the research was to choose the most meaningful model parameters of that given in Tab. 1 and to find any principles that could be helpful in a practical application of the model. The research was realized in a series of numerical calculations of Eq. 1, because the formula depends on reliability function of \( T \) and \( \Delta T \) and its common analytical solution is inaccessible.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Basic value</th>
<th>Test range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_B )</td>
<td>the purchase cost of a new element B</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>( C_R )</td>
<td>the total cost of all activities of: decomposition, cleaning, preparing of the returned B element to reusing in a production</td>
<td>( C_R(T) = \Delta T / 100 ) ( C_R(T) = (\Delta T + T) / 50 \div 200 ) ( C_R(T) = (\Delta T + T) / 50 \div 200 )</td>
<td></td>
</tr>
<tr>
<td>( C_M )</td>
<td>the cost of element’s age identification</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( C_O )</td>
<td>the penalty cost resulting from a product failure during warranty period</td>
<td>5</td>
<td>( 1 - C_B \times 10 \times C_B )</td>
</tr>
<tr>
<td>( T_W )</td>
<td>the warranty period of the object</td>
<td>50</td>
<td>10 ÷ 110</td>
</tr>
<tr>
<td>( T )</td>
<td>the age of the return</td>
<td>-</td>
<td>0 ÷ 100</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>Period which is corresponding to the component age reduction thanks to the implementation of the recovery</td>
<td>-</td>
<td>0 ÷ ( T )</td>
</tr>
<tr>
<td>( R_A(t) )</td>
<td>the A element’s reliability function</td>
<td>( R(t) = e^{-(\alpha_A)t} ) basic: ( \alpha_A = 2, \beta_A = 100 )</td>
<td></td>
</tr>
<tr>
<td>( R_B(t) )</td>
<td>the B element’s reliability function</td>
<td>( R(t) = e^{-(\alpha_B)t} ) basic: ( \alpha_B = 2, \beta_B = 100 )</td>
<td></td>
</tr>
</tbody>
</table>

3.1. COST COMPONENTS ANALYSIS

The first task that was undertaken by the authors was model testing for the unit cost parameters. The goal of the research was to find out which costs components has the strongest impact for the reusing policy in the modeled production system. The results are given in Fig. 5 - 10.
Fig. 5, 6 present the total profit of the reusing policy for various relations of the unit costs $C_O$ and $C_B$. As one might expected, the increase of the warranty cost $C_O$ decreases the profitability of reusing application and force the producer to apply more effective repair actions that reduces significantly the age of the component. All tested relations $C_O/C_B$ have shown that no matter what is the “initial” age of the return, reusing of elements that are restored to be as good as new is always profitable. This obvious fact is always true when as good as new element obtained from return is cheaper than the new one.

The next investigated costs parameters of the model are related to the cost of component rejuvenating (Fig. 7-10). Fig. 7,8 present the total profit of the reusing policy for simple linear
function of $\Delta T$, while results visible in Fig. 9,10 have been obtained for the cost function $C_R$ dependent upon both: $\Delta T$ and age of the return $T$. As one can see, when element’s restoring to as good as new condition is less expensive than a price of a new element, even reusing of “old” components after repair is still profitable (Fig. 8). On the other hand all tested cases when achieving as good as new return is more expensive than a new one (Fig. 7,9,10), the age of reusable components shortens, but this age reduction is not proportional to the change of the cost.

The investigation conducted for costs of element’s identification $C_M$ has proved that even for $C_M = 0.8C_B$, the reusing is still profitable but for little costly returns – “young” elements that do not need any repairs.

Concluding the analysis of the cost components, authors do not compare the strength of the unit costs’ impact on the model results. The obtained outputs present only the directions of profitable solutions and may suggest the actions that may be taken: direct reusing, repair or disposal.

3.2. WARRANTY PERIOD RESEARCH

The second research that was planned and made concerns the length of modelled warranty period. The literature finds this parameter as a key factor when the threshold age of reusable element is determined [9,13]. The aim of planned calculations was testing the variable in order to assess its influence on the economy of the reusing policy.

Fig. 11, 12 present the effect of warranty period length for the total profit of the reusing policy and options. The Fig. 11 shows explicit dependency of the obtained results and warranty period length. The fact of a big discrepancy in costs and savings for “average” values of $T_W$ may be surprising. It suggests that reusing option (direct reusing or repair) has no significance when period is very short and very long – the reusing is always profitable. The middle of the tested $T_W$ range (two chosen cases are presented in Fig. 12) requires more careful decisions regarding the way of reusing options. The explanation regards to the shape of the B component reliability function (Fig. 2) and the fact of a new element’s reliability function comparing with residual reliability of the reused element. Sometimes (for very “young” returns or older element and short warranty periods) it may happen that reused element will have higher reliability during the warranty time than the new one. The explained effect is also the reason of the reusing policy that may be seen in Fig. 11 for $T_W = 10$). On the other hand, when the warranty period is longer than the component’s MTTF (Fig. 11, $T_W = 110$), the reusing is profitable because of low reliability for both: new and reused element. Even reusing of “old” element cannot deteriorate reliability results during warranty, thus the reusing is always profitable.

4. CONCLUSIONS

The model presented in this paper is the continuation of wide researches in the reverse logistics area. The model proposed in this paper is the development of the previous ones presented in
The majority of models deal with single-element system or with the assumption that reused elements are as good as new. The proposed model develops the previous ones by releasing both assumptions. The model is presented and tested for two-element, series system but it is very simple to be developed to the case of x-element, series system. Despite the fact that many companies realize the potential of reusing products, the question "is it worth to do it", is not so simple to answer. The objective of the presented model is to estimate profitability of using returned and recovered elements in a new production, in the case when they are not as good as new. The usage of recovered components decreases production costs but also increases the risk that additional costs occur because of larger amount of returns during the warranty period. Presented model takes also into consideration alternative action - element's rejuvenating with additional cost.

The practical application of the presented model requires the verification conducted on the base of statistical data coming from companies that already apply the reusing policy. The efforts taken by the authors aims at gathering the sufficient database in order to confirm the theoretical model and numerical results.

Corresponding to conclusions in [9, 13] the practical application of the proposed model is limited because of the mentioned simplification that all reused elements are in the age of T. According to model assumptions, the demand for the products is determined and fixed. As in [9] the practical application of proposed model require more precise data: how many reusable elements will return before the new production batch, how many new components must be kept in a stock or what is the expected profit/cost when mix of new-old elements is used in the production. To answer this questions, the model should consider the percent of returns used in the production. The number of returns that can return between two moments of a production beginning and may be reused depends on the number of the products that were sold earlier, the length of the period between two consecutive production batch, the length of the warranty period.

5. REFERENCES

[15] Sherbrooke C. C., An Evaluator for the Number of Operationally Ready Aircraft in a Multilevel


