Logistics Systems Maintenance Modelling Problems - the Use of TDA Approach

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The article presents an overview of some recent developments in the area of mathematical modelling of maintenance decisions with the use of time-delay and time-delay analysis. Thus, the literature overview in the investigated area is provided. The problem of time relations occurred in supply chain performance process is investigated. Later, there are presented obtained reliability analysis results in comparison with the knowledge about the case company present condition. The article investigates the possibilities of basic time delay model implementation in the area of logistic system of sixteen forklifts performance analysis.

**Keywords:** time-delay analysis, logistics system performance, inspection model

1. INTRODUCTION

Any logistic system, operating under an increasingly complex and diverse system environment, may fail what in consequence may lead to [62]:

- disruption of supporting task realization,
- inability of system to undertake a new task.

Thus, there is a need to take into account the possible unreliability of logistic system elements, which may lead to decrease of the system availability level. In the maintenance and reliability theory literature, there are many studies dealing with the problem of designing reliable and available logistic support systems for repairable items performance [61]. However, most of the developed models focus only on spare parts availability (see e.g. [13, 30]) and repair facilities availability (see e.g. [1, 9, 35, 50, 51, 58, 59]). The problems of possible interactions, which may occur between operational system and its logistic support system, usually are not analyzed [61].

To overcome this problem, “system of systems” conception is proposed to model the interactions between both systems being in cooperation. The defined conception gives also the possibility to define the main time relations between the systems. More information can be found in e.g. [61, 62, 63, 64].

The second approach refers to time-delay approach implementation, which is relevant to this paper.

Following the introduction, this paper is focused on time delay modelling. Consequently, the paper is organized as follows: in the Section 2, a literature overview of the most often applied time-delay models is presented. The basic model of inspection based on time delay modelling is also provided. Later, the issues of time relations occurred in logistics systems are investigated. On this background, in the Section 4, the obtained reliability analysis results are presented in comparison with the knowledge about the case company present condition. The analysis is performed for sixteen forklifts, which are operated and maintained by the case company. Author focuses on the problem of time delay parameter estimation. Later a briefly summary is provided.
2. TIME-DELAY APPROACH - LITERATURE REVIEW

In the case of complex systems, the problem of relations between two consecutive subsystems modelling occurs. One of the concepts which provide useful means of modelling the effect of periodic inspections on the failure rate of repairable technical systems is a time delay concept, developed by Christer et al., see e.g. ([18, 19, 23, 24, 25, 28]).

The time delay concept defines a two-stage process for a component. First, a fault which has developed in the system becoming visible at time \( u \) from new with probability density function, pdf \( g_u(u) \), if an inspection is carried out at that time. If the fault is not attended to, the faulty component fails after some further interval \( h \) which is called time delay of fault and is described by probability density function, pdf \( f_h(h) \) (Fig. 1). During the period of \( h \) there is an opportunity to identify and prevent failure.

![Figure 1. Time delay modelling concept [45]](image)

Once these two distributions are known, it is possible to model the reliability, operating costs and availability. The variables \( u \) and \( h \) depend upon the inspection technique adopted, as described in papers [18, 22, 23, 24].

In general, there are two methods to estimate these model parameters, namely the subjective method and the objective method. The first one is based on subjective data obtained from maintenance engineers’ experience (see e.g. [23, 24]). The second one is based on the objective data of recorded failure times and the number of defects identified at Preventive Maintenance (PM) (see e.g. [6, 7]). Moreover, in the 1990s, models were published where the two model parameters \( u \) and \( h \) are estimated using limited PM data and selective repair at PM (see e.g. [27]).

Time delay modelling applied to industrial maintenance problems can be traced back to [19]. In this paper author investigates the model of inspection policy for building complex based upon information likely to be available and subjective opinions. Since then, there has been increasing effort to encourage the use of the time-delay concept in maintenance theory. Many works have been carried out on the modelling of this concept to production plants (e.g. [2, 21, 23, 26, 27]). Other works include the application of failure process to gearbox (e.g. [43]), modelling PM for a vehicle fleet (e.g. [24, 33]), PM process for a coal face machinery (e.g. [14]), wind turbine maintenance optimization (e.g. [4]), modelling ship operational reliability (e.g. [20]), and application to concrete structures (e.g. [22, 48]). Moreover, well known models base on time-based PM (e.g. [46]) or condition-based maintenance (e.g. [17]) implementation. Additionally, Cui in his work [31] provides a semi-automated TDM (time delay modelling) tool, which is based on models developed till 2002. Developed tool enables engineers to undertake primary data analysis, model building, model testing, and model based decision making.

The main questions being investigated in such works are [16]:

- How often vehicles should be maintained preventively or repaired/replaced?
- Is a reliability structure of a system safe?
- How often should production plant be maintained periodically?
- What is an economical and technical effectiveness of a performed maintenance?

A literature review, in which time-delay models are investigated along with other PM models are given in [32, 36, 37, 44, 53, 55]. The state of art works, dedicated to TD modelling, is mostly developed in the 1990s. According to work [16], one of the first publications which strictly investigated recent development in the time-delay modelling is given by Christer & Redmond [22]. In the mentioned paper, the basic TD model for a single unit case is provided. Moreover, problems of model parameters estimation are discussed. Later, the state of art is updated in [21]. In this
work, mathematical methods for updating time-delay models of industrial inspection practice are proposed.

In 1993, Alzubaidi [3] investigates the known Operational Research modelling models in the area of:

- management problems,
- overhauls,
- inspection models, including TD modelling problems,
- preventive maintenance,
- capital equipment replacement, and stochastic maintenance and replacement.

The author focuses on the problems of building maintenance. In this context, the TDA was introduced as a convenient solution for modelling the consequences of an inspection policy for industrial inspection maintenance. The author presents the basic time delay model with some variations (like e.g. imperfect inspection performance).

Later, in [5] authors discuss the development of time-delay analysis as a means of modelling engineering aspects of maintenance problems. In this work, authors present the idea of TD modelling for multi-component systems. The basic assumptions, common for all the known TD models, are given with the definition of some additional assumptions being widely used in this research area.

In the next work [47], the author provides a recent development in TD modelling issues. The state of art classifies the known models from several points of view. At the same time, the TD modelling literature review, taking into account the possibility of using Semi-Markov Inspection models for single component and multi-component systems is provided in [15].

Christer in [16] reviews the recent cumulative knowledge and experience of time delay modelling. Author presents the basic time delay model and discusses the main development directions including:

- perfect/non-perfect inspection,
- steady state and non-steady state conditions,
- type of arrival rate of defects, or
- type of model’s parameters estimation process.

The investigation of known TD models is also proposed in [42], where author focuses on the maintenance practice of complex plant. The literature review of single component TD models and complex system TD models is also discussed in [31].

To sum up the above investigations, the main classification criteria for TD models are the following [5, 16, 22, 45, 47]:

- single-/multi-unit case,
- perfect/imperfect inspection,
- known/unknown time delay parameter,
- method of parameters estimation,
- finite/infinite time horizon,
- optimization criteria, and type of maintenance policy (e.g. time-based/condition-based/RCM-based).

However, the known models have an arbitrary nature of their assumptions, e.g. [16, 22, 45]:

- independent system elements,
- infinite time horizon,
- perfect replacement/inspection policy,
- omitted reliability structure of a system,
- failure rate and repair rate independent on components age,
- cost criteria usually used to find maintenance policy parameters.

Some of these problems are investigated in [38, 39, 40]. In work [38] authors investigate the effectiveness of a system when defined maintenance strategy is one of two kinds. First, the typical group maintenance policy is considered. Second one refers to time delay model. Authors assume that the system is a three-component with a k-out-of-n reliability structure. To make a comparison, the basic assumptions are defined to be the same for both investigated models. Next, in [39] authors focus on investigation of Block-Inspection Policy performance level with economical and availability point of view. Later, in [40] authors investigate the issues of time delay parameter estimation.

Following these considerations, in the Fig. 2, there is presented the main classification of existing time delay models.
In the presented scheme, three main groups of time-delay models are defined according to the maintenance strategy used. First group is devoted to time-based inspection models. There, plenty of studies refer to block-replacement inspection policy, where inspection takes place every $T$ time units. The second group of models which deserve to be mentioned are condition-based maintenance models. There one can find models especially developed for production plant maintenance. The last group of models introduce the time delay concept usage in reliability centered maintenance. However, there are only a few works which refer to this maintenance area.

Moreover, when analysing the time relations modelling problems, there is also a necessity to investigate the issues of logistics delays occurrence during defence systems operational processes performance [49]. The main problem in this area is connected with definition how logistics processes performance delays affect unreliability (total downtime) of a supported system (e.g. maintenance time delay connected with spare parts delivery delays), and as a result, the main system dependability characteristics [61].

The presented defence approach may also be used in time analyses of production and technical systems performance, connected with [61]:
- delays occurred during operational processes performance (between consecutive operations),
- random lead times,
- spare parts availability.

The convenient example, which serves to illustrate the complexity of such problems is given in [61], where the system of systems with time resource model is developed.

### 2.1. Basic Model of Inspection

One of the main inspection policy models, which are based on the TDA approach implementation, is given in [23]. The investigated simplest possible case of an inspection policy is characterized by the following assumptions [23]:
1. there is a constant time $T$ between successive inspections which require $d$ time units,
2. inspection costs $I$ units,
3. inspections are perfect in that any defect present within the plant will be identified,
4. inspections are independent of each other,
5. faults are independent and arise within the technical system at a constant rate \( k \) for any inspection period,
6. time of origin of the fault is uniformly distributed over time since the last inspection,
7. defects identified at an inspection will be repaired within the inspection period,
8. breakdowns impose a small amount of downtime, \( d_b \), compared to the inspection interval \( T \) and \( d \),
9. the time delay of a fault is independent of the arrival time and has known pdf \( f_h(h) \) and cdf \( F_h(h) \).

For such assumptions it is possible to estimate the probability of a fault arising as a breakdown \( P_b(T) \):

\[
P_b(T) = \frac{1}{T} \int_0^T (T - h/T)f_h(h)dh
\]

The expected downtime per unit time to be incurred operating an inspection policy of period \( T \) is given by \( E_d(T) \), where:

\[
E_d(T) = \frac{kTd_bP_b(T) + d}{T + d}
\]

With average breakdown and inspection repair costs \( c_b \) and \( c_i \) respectively, the expected cost per unit time of maintaining the plant on an inspection system of period \( T \) is \( C(T) \), where:

\[
C(T) = \frac{1}{(T + d)}\left[kT[c_bP_b(T) + c_i(l - P_b(T))] + 1\right]
\]

Let’s consider the second case, when the inspections are non-perfect. Thus, there is introduced a probability \( \beta \) that a specific defect will be identified at \( n \)th inspection, and a corresponding probability \( (1 - \beta) \) that it will not. For such an assumption the modified form of \( P_b(T) \) is given by:

\[
P_b(T) = 1 - \left\{ \int_0^T \sum_{n=1}^\infty \frac{\beta}{n!} R(nT - y)dy \right\}
\]

There are many variations of the presented model, being investigated in known literature from reliability theory. Moreover, one can find few works in which the described model is used to real-life systems’ reliability analysis performance. For example see [45].

3. TIME RELATIONS IN LOGISTICS SYSTEMS PERFORMANCE

According to the definition developed by the Council of Logistics Management [10], logistics is that part of supply chain process that plans, implements and controls the efficient, effective forward and reverse flow and storage of goods, services and related information between the point of origin and the point of consumption in order to meet customers’ requirements. As a result, the components of a typical logistic system are [8]: purchasing, traffic and transportation, warehousing and storage, customer service, demand forecasting, inventory control, material handling, order processing, parts and service support, packing, return goods handling and informational flows performing. Logistic processes are performed by using resources in the form of: equipment, manpower, facilities, and financial assets [62].

There are many models in the literature which are concerned with material procurement, production, transportation, storage or distribution activities. However, lot of them treat each stage of supply chain as a separate system [29]. As a result, many of the supply chain interactions are ignored. This may lead to improper identification of elements, which may influence the proper performance of a given chain (Fig. 3).
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Figure 3. Reliability, availability, maintainability and supportability in achieving supply chain performance [52]

Supply chain networks are vulnerable to disruptions and failure at any point in the supply chain may cause the entire network to fail. A key factor in effective supply chain management is the ability to minimize the effects of such undesired events/disruptions occurrence. As a result, understanding what disruptions may occur in a supply chain, how they will affect a supply chain system, and how far reaching these effects will be, would be of considerable benefit [65].

Treating the supply chain disruptions as unexpected events occurrence, we can describe them as having uncertainty in supply chain operations [65]. Uncertainty in the supply chain can be seen from different aspects, such as [56]:

- time (in the sense of duration of activity/process, starting/ending moment of activity realization, frequency of activity/demand occurrence),
- quantity (of supply, demand or physical transfer of goods),
- location/place (where activity starts/ends),
- quality (of service/products),
- cost (fluctuation, occurrence).

However, not every disruption occurrence leads to logistic system failure appearance. The critical factor which determines the logistic system failures is time. In a situation, when disruption (connected with e.g. improper delivery quality/quantity, improper location) occurs, there is a necessity to find out if we have enough time to correct the problem. When the spare time lets us remove the disruption – logistic system is not defined as failed. In other words, time redundant system has the ability to tolerate interruptions in their basic function for a specific period of time without having the negative impact on the system task performance.

Typically, the time redundant systems have a defined time resource, denoted by $\gamma$ that is larger than the time needed to perform the system total task. However, unreliability of system element may cause time delays which in turn would cause the system total performance time to be unsatisfactory. As a result, considering the system task completion time as a random variable, the probability that mentioned time will be longer than the restricted time resource may be defined as the unreliability index [41, 60, 63].

According to the present knowledge, the time redundancy is considered as the effective tool for e.g. reliability improvement [54, 60]. In the known literature, there are many works which investigate the problem of time redundancy (see e.g. [11, 12, 54, 60]). Moreover, taking into account the following issues [54, 60]:

- type of failure tolerance,
- type of time resource usage,
- type of time resource replenishment,

one can define the main types of time resource, which are described e.g. in [63]. The proper type of time resource selection depends on the type of modeled system, type of operational task, efficiency definition, and system reliability structure. The main models classification of systems with time resource is given in [63].

4. CASE STUDY – IMPLEMENTATION OF BASIC MODEL OF INSPECTION BASED ON TDA

To model time relations in logistic system performance, the basic model of inspections is used described in the Section 2.1. The investigated model focuses on logistic system elements functions and defines the optimal time between inspections $T$, which allows system to perform satisfactorily.

The basic TDA model implementation is analyzed based on the information about operational and maintenance processes of sixteen forklifts, which are operated by considered case company [34]. The gathered data, used in reliability analysis regard to eight years of trucks performance time period, since 2000 till 2008. The performance data include operational processes realization, repair and preventive maintenance times, types of replaced elements, and types of occurred failures. The shortest mileage of analyzed forklifts is 2 800 working hours and the longest mileage is equal to 13 300 working hours.
4.1. MAINTENANCE TASKS BEING PERFORMED DURING ANALYZED TIME PERIOD

In the first step of performed reliability and maintenance analysis, author gathered the data about maintenance tasks (replacing of elements, spare parts, repairs), being performed during analyzed time period. The gathered information is classified into seven working groups:

- track frame and body,
- installation and electrical equipment,
- hydraulic circuit,
- lifting circuit,
- transmission,
- steering system,
- brake system.

In the Table 1, the main maintenance tasks performed in lifting circuit during analyzed time period are presented.

<table>
<thead>
<tr>
<th>Maintenance task</th>
<th>Maintenance actions’ object</th>
<th>Number of performed maintenance actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>replacement</td>
<td>mast rollers</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>mast support bearing</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>lifting chain</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>forks lock</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>lifting chain safety device</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>forks</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>mast roller</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>tilt cylinder’s pins</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>tensioning screw</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>mast fixing screw</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>tilt cylinder</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>lifting cylinder</td>
<td>1</td>
</tr>
<tr>
<td>Repair</td>
<td>tilt cylinder</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>broken lifting chain</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>mast rollers</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>lifting cylinder (sealing)</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>truck mast (welding, grinding)</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>lifting chain regulation</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>regulation of lifting and tilting functions</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>lifting chain’s rollers cover</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>forks security bars</td>
<td>4</td>
</tr>
</tbody>
</table>

The trucks inspection actions are performed according to the given operators manuals. During the inspection performance it should be checked out if all circuits and transmission perform satisfactory, especially:

- security systems,
- forks and carriage,
- steering system,
- lighting system,
- signalling system,
- properness of performed maintenance actions,
- frame (every 12 months).

Preventive maintenance actions are performed according to the trucks mileage indications. Moreover, inspections are performed after every 1000 working hours (e.g. mast, forks and axle maintenance), after every 2000 working hours (e.g. lifting system maintenance), or after every 3000 working hours (e.g. hydraulic system maintenance).

4.2. MAIN RELIABILITY CHARACTERISTICS OF ANALYZED SYSTEM’S ELEMENTS

Reliability analysis of forklifts (repairable objects) has been performed based on the data from service manual where the information about trucks actual mileage, types and times of performed maintenance actions can be found. Thus, the obtained data let the author estimate the main reliability characteristics of the chosen trucks. The data analysis has been carried out with the use of Weibull ++ software, what gives the possibility to estimate e.g. probability functions of times to failure or repair times. The chosen probability characteristics of times between failures are given in Fig. 4-5, and for repair times given in Fig. 6. The defined functions are a lognormal probability functions. The main trucks operational parameters are given in the Table 2.
4.3. BASIC MODEL OF INSPECTION IMPLEMENTATION

The implementation of TD model of inspection needs the main model parameters estimation:

- inspections time \( d \) is equal to 2 hours,
- downtime \( d_b \) equals 5.18 h (estimated as a mean repair time of all sixteen trucks in a given time period),
- total operational time of all trucks in a given time period is 131700 working hours,
- the constant rate \( k \) of fault occurrence is estimated at the level \( k = 0.006348 \) per hour (836 failures during analyzed time period),
- MTBF equals 16 working hours, with standard deviation being equal to 26 working hours.

The formula given in (2), when using formula (1), may be defined as:

\[
E_d(T) = \frac{kT \left[ \frac{1}{T} \int_0^T (T-h)f_b(h)dh \right] d_b + d}{T + d}
\] (5)
Substituting obtained values of model parameters into equation (5) gives the following:

\[
E_d(T) = \frac{(0.006348T) \left[ \frac{1}{T} \int_0^T (T-h)f_s(h)dh \right]}{T + 0.5} \cdot 5.18 + 0.5
\]

The last problem, before model functions estimation, is time delay distribution definition. This distribution function can be estimated using two methods, namely subjective and objective ones. In the known literature, few models have been developed for these two approaches, for more information see e.g. [5, 6, 65]. The use of objective models requires a large amount of data in comparison with survey questionnaires which should reflect the operations of the analyzed system over a considerable period of time. These requirements usually are difficult to fulfill when considering a real-life system performance.

When dealing with the analyzed forklift system, there is a problem to choose the most appropriate distribution function for \( h \) parameter. As a result, there is a necessity to study different pdf functions of time-delay. The functions of exponential distribution have been investigated, Weibull distribution, and standard normal distribution.

First, the exponential distribution for the time delay is used, where:

\[
f_h(h) = \lambda e^{-\lambda h}
\]  

Substituting formula (7) into Equation (6) to obtain an expression for the downtime will give:

\[
E_d(T) = \frac{(0.006348T) \left[ \frac{1}{T} \int_0^T (T-h)\lambda e^{-\lambda h}dh \right]}{T + 0.5} \cdot 5.18 + 0.5
\]

The Weibull distribution for the time delay parameter is given by:

\[
f_h(h) = \frac{\alpha}{\beta} h^{\alpha-1} e^{-\left(\frac{h}{\beta}\right)^\alpha}
\]  

Thus, the Equation (6) gave the following:

\[
E_d(T) = \left\{ \frac{1}{T + 0.5} \right\} \cdot 0.5 + \\
+ (0.006348T) \left[ \frac{1}{T} \int_0^T (T-h)\left(\frac{\alpha}{\beta} h^{\alpha-1} e^{-\left(\frac{h}{\beta}\right)^\alpha} \right)dh \right] \cdot 5.18
\]  

When the pdf of the time delay follows the normal distribution:

\[
f_h(h) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(h-\mu)^2}{2\sigma^2}}
\]

where: \( \mu \) = mean of \( h \), \( \sigma^2 \) = standard deviation of \( h \)

Let \( \mu = 0 \) and \( \sigma^2 = 1 \), assuming standard normal distribution, Equation (11) is simplified to become:

\[
f_h(h) = \frac{1}{\sqrt{2\pi}} e^{-\frac{h^2}{2}}
\]

Substituting Equation (12) into Equation (6) gives:

\[
E_d(T) = \left\{ \frac{1}{T + 0.5} \right\} \cdot 0.5 + \\
+ (0.006348T) \left[ \frac{1}{T} \int_0^T (T-h)\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{h^2}{2}} \right)dh \right] \cdot 5.18
\]  

The results are given in Figures 7-9.

The results obtained using exponential distribution are not very useful, because they do not reflect a curve that increases in \( E_d(T) \), as the inspection period increases. From these results, the most suited distribution was found to be the Weibull and standard normal distribution. These two distributions give the indications of the optimum inspection period.

Figure 7. Function \( E_d(T) \) when using exponential distribution for time delay \( h \)
5. SUMMARY

The time delay concept is a convenient one within the maintenance engineering context. This concept can be used to build different qualitative models of the inspection practice of different real-life systems like production plant, transportation systems or civil engineering systems. These models’ attention is focused on what to do, as opposed to the logistical decisions of how to do.

However, there is a problem with reliable data obtaining. There exist in the literature three main techniques to estimate the time delay parameters given objective data, subjective data or a mixture. Set of known works with applications have given consistent results. But, author puts an emphasis on problems with gathering well-tested methods to collect adequate data to enable the calculation of optimum maintenance policies for dedicated systems.

The last problem regards to assumptions of maintenance policy parameters estimation. The well-known models assume exponential approximation of model parameters’ distributions. It is convenient situation from a modelling point of view, but not sufficient to real-life systems performance.

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