The Use of Stochastic Stock Model for Determination of the Optimal Quantity of Supply

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This article deals with the use of stochastic model for determination of the optimal quantity of supply of supplier to automotive industry. Today manager's practice emphasizes the decrease of stocks. The main issues considered in this topic are: the mathematical stochastic stock model to determine the optimal quantity supply.
Keywords: stock, inventory management, warehouse, cost, supply, abc analysis, company

1. INTRODUCTION

This article deals with the inventory management and method of determining optimal quantity of supply of supplier to automotive industry. Nowadays automotive industry develops in the Czech Republic. The modern logistic methods (JIT, KANBAN, KAIZEN etc.) are used in the branch of automotive industry. These methods require a lot from suppliers. Main requirements are: quality of supplied components, in-time supplies and other.

2. IMPORTANCE OF STOCKS FOR COMPANY

The importance of stocks for company is great. Today manager's practice emphasizes the decrease of stocks. Negative influence of stocks is that they tie the capital, use up work and means (stocks have to be stored, consequently: energy costs, service costs, costs of repairing, labour costs, etc.), risk of stock depreciation, risk of unserviceableness or risk of unsaleability (possible reason: change of production program or customer preference). Locked-up capital in stocks is short of the technical development financing, it endangers ability to pay to company and decreases trustworthiness during meetings with business partners and banks (high-stock level means bad business management). The stock level should be decreased to the minimum but on the other hand it has to ensure sufficient readiness of supplies to customers. It is evident, that both points of view (stock minimization versus high readiness of supplies) are opposed and the company has to choose the compromise.

3. CLASSIFICATION OF STOCKS

Each production company has its classification of stocks. Stocks can be classified according to many points of view (for example: degree of processing, suitability, function, etc.). Function classification of stocks is the most important from the point of view of operative management.

Function classification of stocks:

- turnover stock
- strategic stock
- safety stock
- technological stock
- seasonal pre-stock
- speculative stock

Turnover stock is the part of stocks which ensures that requirements are met between the two supplies. Its level fluctuates during the supply cycle. For that reason it works with the average turnover stock. Its level is half of the supply in ideal case.

Strategic stock ensures functioning of company during unpredictable incidents (calamity, strike, conflict, etc.).

Safety stock has the aim to balance stochastic fluctuations partly on the input side to the company (level of supplies and supply interval), partly on the output side from the company (level and interval of draw on stocks).

Technological stock is kept in the event production is terminated by a producer, but the product cannot meet consumer demand because it needs to be stored before application. Some foods (cheese, beer and Wine) have to mellow more time.

Seasonal pre-stock balances expected fluctuations in input or output. Because of limited production capacity, in some periods companies are not able to satisfy customer's demand for some seasonal products (e.g. at Christmas time). In this case the company begins to make planned stock of a product in advance. The company assumes that this planned stock will sell.

Speculative stock is kept in order to achieve extra profit with suitable purchase (temporary reduced prices before expected increase of a prices, purchase of advantageous future sale).

4. ECONOMIC ORDER QUANTITY (EOQ) MODEL

The best ordering policy can be determined by minimizing the total of inventory carrying costs and ordering costs using the economic order quantity (EOQ) model (Grant et al., 2006).

We can find this model described in a book which is called 'Inventory Management' (EMMETT, 2008) too.

The EOQ model is a known concept which determines the optimal order quantity on the basis of ordering and carrying costs. Following Fig. 1 illustrates EOQ model.

The EOQ in units can be calculated using the following formula:

$$EOQ = \sqrt{\frac{2 \cdot P \cdot D}{C \cdot V}}$$

where:
P - the ordering cost per order
D - annual demand or usage of product (number of units)
C - annual inventory carrying cost (as a percentage of product cost or value)
V - average cost or value of one unit of inventory

The EOQ model has received significant attention and application in industry, but not without its limitations. The simple EOQ model is based on the following assumptions:

- a continuous, constant and known rate of demand
- constant and known replenishment or lead-time
- constant purchase prise independent of the order quantity or time
- constant transportation cost independent of the order quantity or time
- the satisfaction of all demand (no stock-outs are permitted)
- no inventory in transit
- only one product in inventory or at least no interaction between products (independent demand items)
- no limit on capital availability

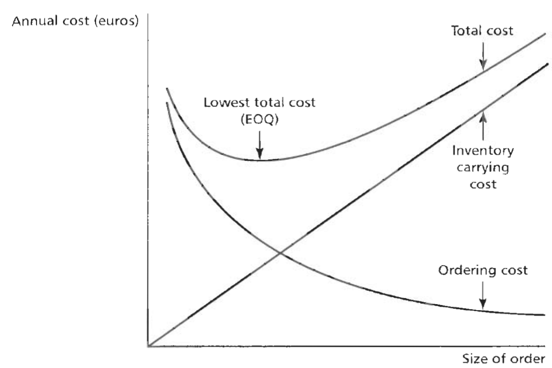

Figure 1: EOQ model

5. DIFFERENTIAL INVENTORY MANAGEMENT

Paret's rule (author Villefredo Pareto (1848-1923)) deals with differential inventory management. In practise it isn't possible, not useful; to pay the same attention to all stock items. For that reason it is necessary to differentiate between various inventory managements. Paret's rule helps to determine the most important stock items. Paret's rule says that 80 % consequences result from 20%

possible causes. In case of stock it means that 20 % of stock items can represent 80 % value of consumption or sale or big part of purchase originates from relatively small number of suppliers. Paret's rule means that inventory management should focus on certain number of the most important objects (for example: stock items, suppliers), which influence the total result.

On the basis of Paret's analysis we can classify stock assortment to groups according to some criteria. In practice, stock items are classified to three groups. This analysis is called ABC analysis. ABC analysis is described in many books (for example: *Fundamentals of Logistics Management, 2006*)

The inventory management has to arrange stock items downwardly according to value of monitored statistical index (value of consumption or value of sale) in monitored time. Analysed period should be from 12 to 24 months. Shorter period can result in misrepresenting due to seasonal influence. In a longer period production program may be changed and data on capability may be lost. The next step is to find the items which represent 80 % and 90 % value of consumption or value of sale.

Category A consists of stock items which make 80 % value of consumption or value of sale. They are the most important stock items which should be monitored daily. The optimal supply and safety stock is determined individually and as exact as possible.

Category B represent stock items with 15 % value of consumption or value of sale. It means that both categories A and B represent together 95 % value of consumption or value of sale. Stock items of category B are monitored as compared with category A less often and simpler methods are used to manage them. Value of supply and safety stock is usually higher than stock items of category A.

Category C represent stock items with approximately 5 % value of consumption or value of sale. Very simple methods are used to manage them. These methods use estimation of average consumption in preceding period.

6. STOCHASTIC STOCK MODEL

We use stochastic stock model which was proposed by authors Svoboda and Latyn (2003).

We will suppose that intensity of warehouse collection - meaning value of withdrawal pieces per unit time is λ. Stocking products arrive to the warehouse in the quantity of *m* pieces which are the most numerous products in the warehouse. If *K* is total warehouse capacity then $m=K$. Intensity of supply is μ. An empty warehouse triggers supplies – consequently empty stocks are always replenished.

The time between warehouse collections is the inverse value of intensity of warehouse collection $1/\lambda$, similarly the value between two supplies is the inverse value of intensity of supplies $1/\mu$.

Thanks to queuing theory we derive transition probability. Transition probabilities mean that the system goes over from system state *i* to system state *(i+1)* or *(i-1)* or stays in system state *i* (P. M. Morse in work 'Queues, Inventories and Maintenance' deals with model).

Probabilities p_i of system state *i* (the system has just *i* system states) were derived on condition of stationary distribution – equations (1) and (2).

$$p_1 = p_2 = ...= p_K = \frac{\mu}{\lambda} p_0 \qquad (1)$$

$$p_0 = \frac{1}{1 + K\frac{\mu}{\lambda}} \qquad (2)$$

We will optimise quantity $m=K$, on condition that profit is the criterion of optimization. We will assume that profit per sold unit is *g*, hold costs per time *T* on unit are c_1 and costs of transposition supply $m=K$ units are c_0. If *K* is high enough, it is possible that the continuous function approximates development of profit function - equation (3).

Profit function is:

$$Z(K) = g \cdot L_s - c_1 - c_0 p_0 \qquad (3)$$

where:

$$L_s = T(1 - p_0) = \frac{\lambda \cdot T \cdot K}{K + \lambda \cdot T} \qquad (4)$$

L_s is average number of warehouse collections per time T.

We can calculate the average stock:

$$I = \sum_{n=0}^{K} n \cdot p_n = \frac{K^2 + K}{2(K + T\lambda)} \qquad (5)$$

After appointment into equation (3), we get:

$$Z(K)=\frac{g\cdot T\cdot\lambda\cdot K-\frac{c_1}{2}\cdot(K^2+K)-c_0\cdot T\cdot\lambda}{K+T\cdot\lambda} \quad (6)$$

First derivative of profit function (6) is zero for the determination of maximum profit and then we can deduce level of supply:

$$K_{1,2}=-T\cdot\lambda\pm\sqrt{(T\cdot\lambda)^2-} $$

$$-T\cdot\lambda+\frac{2c_0}{c_1}\cdot T\cdot\lambda+\frac{2}{c_1}\cdot g\cdot(T\cdot\lambda)^2 \quad (7)$$

Because the positive solution is convenient to the problem and other quantities are negligible in relation to c_0/c_1 a g/c_1. it is possible to simplify the equation (7). We will assume that minimum size of warehouse equals the optimal supply. We can determine level of supply m_0 and warehouse capacity as follows (8):

$$m_0=K_0\approx\sqrt{2T\lambda\cdot\frac{g\cdot T\cdot\lambda+c_0}{c_1}} \quad (8)$$

In case that criterion of optimization is to minimize costs so cost function is following (9):

$$N(K)=\frac{\frac{1}{2}c_1\left(K^2+K\right)+c_0T\lambda}{K+T\lambda} \quad (9)$$

Optimal level of supply (N´(K)=0) will be in the same case as optimal criterion of profit (10):

$$m_0^{'}=K_0^{'}\approx\sqrt{\frac{2T\lambda c_0}{c_1}} \quad (10)$$

Till now we considered an empty warehouse, consequently the empty stock is always replenished. In practice it isn't always possible. We have to define number of units in the warehouse, where stock level mustn't drop below safety stock *D*. Impulse for the supply will be reaching safety stock. Then we order m= K-D units to replenish stock in the warehouse on level *K*.

We deduce forms of probabilities for the stationary distribution of system probabilities (with the help of queuing theory).

$$2\mu p_0=\lambda p_1 \quad (11)$$

$$0<n\le D$$

$$(\mu+\lambda)p_n=\lambda p_{n+1} \quad (12)$$

$$(\mu+\lambda)p_D=\mu p_0+\lambda p_D \quad (13)$$

$$D<n\le m$$

$$\lambda p_n=\lambda p_{n+1}+\mu p_{n-m} \quad (14)$$

$$m<n\le K$$

$$p_K=\mu p_K-m \quad (15)$$

$$\sum_{n=0}^{K}p_n=1 \quad (16)$$

If is $\mu=1/T$, we can determine probabilities of system state:

$$p_n=\frac{2}{T\lambda}\left(1+\frac{1}{T\lambda}\right)^{n-1}p_0 \quad (17)$$

$$0<n\le D$$

$$p_n=\frac{2}{T\lambda}\left[\left(1+\frac{1}{T\lambda}\right)^{D}-\frac{1}{2}\right]p_0 \quad (18)$$

$$D<n\le m$$

$$p_n=\frac{2}{T\lambda}\left[\left(1+\frac{1}{T\lambda}\right)^{D}-\left(1+\frac{1}{T\lambda}\right)^{n-m-1}\right]p_0 \quad (19)$$

$$m<n\le K$$

$$p_0=\frac{(T\lambda)^{D+1}}{2m(1+T\lambda)^{D}+(T\lambda+D-m)(T\lambda)^{D}} \quad (20)$$

Assuming profit function (6) we can determine optimal level of supply m_0, if first derivative of profit function is zero according to *m*. If the warehouse capacity is K then K_0=K-D is the optimal warehouse capacity over safety stock.

7. SIMULATION

This model of determination of optimal quantity of supply was used in the company. Numbers are fictitious and illustrative for confidentiality reason.

condition m = K

m = K= 5000 pcs.

T =1 day

λ = 100 pcs./day

μ = 150 pcs./day

g = 1200 Kč

c_1 = 35 Kč/day

c_0 = 15 Kč

1) We will optimise quantity m=K, on condition profit is the criterion of optimization.

$$m_0 = K_0 \approx \sqrt{2T\lambda \cdot \frac{g \cdot T \cdot \lambda + c_0}{c_1}} =$$

$$= \sqrt{2 \cdot 1 \cdot 100 \cdot \frac{1200 \cdot 1 \cdot 100 + 15}{35}} =$$

$$= \sqrt{\frac{24000015}{35}} = \sqrt{685714{,}71} = 828\, pcs.$$

2) We will optimise quantity m=K, on condition minimization of hold costs is the criterion of optimization.

$$m_0^{'} = K_0^{'} \approx \sqrt{\frac{2T\lambda c_0}{c_1}} = \sqrt{\frac{2 \cdot 1 \cdot 100 \cdot 15}{35}} =$$

$$= \sqrt{\frac{3000}{35}} = \sqrt{85{,}71} = 9{,}26 \approx 10\, pcs.$$

$$m_0^{'} = K_0^{'} \approx \sqrt{\frac{2T\lambda c_0}{c_1}} = \sqrt{\frac{2 \cdot 1 \cdot 100 \cdot 15}{35}} = \sqrt{\frac{3000}{35}} = \sqrt{85{,}71} = 9{,}26 \approx 10\, pcs.$$

Next conditions which we must take into consideration during decision about order quantity:

- minimum quantity v packing
- minimum order quantity determined by supplier
- according to settings of criterions in ABC analysis

8. CONCLUSION

The inventory management is very important for the production company. The stock level should be decreased to the minimum but on the other hand it has to ensure sufficient readiness of supplies to customers. It is evident that both points of view (stock minimization versus high readiness of supplies) are opposed and the company has to choose the compromise. We used the mathematical stochastic stock model to determine the optimal quantity of supply. Authors warn about factors which the purchaser must take into consideration during decision about order quantity.

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