An Analysis of Stochastic Inventory Control Models in Reverse Logistics Systems Based on a Continuous Review

Marcin Plewa
Wrocław University of Technology, Poland

In reverse logistics systems demand can be partially satisfied with new items manufacture or procurement and returned products value recovery. The products are brought back to the places where they are stored in most models presented in literature on reverse logistics. Value recovery processes are carried out in due time in order to meet the existing demand. Some part of returns can be disposed of.
Inventory management has significant meaning in reverse logistics. This article's purpose is to present models being modifications of a classical inventory control model in a continuous review system. The first model of that kind was developed by Heyman in 1977. Guided by similar assumptions, Muckstadt, Korugan, Fleischmann and van der Laan, among others, designed continuous review models as well.

1. BASIC NOTATIONS

- $D(t)$ – quantity demanded for finished products at time .
- $Z(t)$ – quantity of returns at time .
- T – planning horizon scope.
- $I_{poz}(t)$ – on-hand inventory state in the finished products warehouse at time .
- $I_z(t)$ – inventory current state in the returns warehouse at time .
- $I_{np}(t)$ – inventory current state in the finished products warehouse at time .
- Q_{zam} – optimal order batch quantity.
- $Q_{zam}(t)$ – order batch quantity being underway at time .
- $TQ_{zam}(t)$ – cumulative order quantity at time .
- Q_{prod} – optimal production batch quantity.
- $Q_{prod}(t)$ – production batch quantity being underway at time .
- $TQ_{prod}(t)$ – cumulative production quantity at time .
- Q_{odz} – optimal recovery batch quantity.
- $Q_{odz}(t)$ – recovery batch quantity being underway at time .
- $TQ_{odz}(t)$ – cumulative recovery quantity at time .
- Q_u – optimal disposal batch quantity.
- $Q_u(t)$ – disposal batch quantity being underway at time .
- $TQ_u(t)$ – cumulative disposal quantity at time .
- Q_t – transportation batch quantity.
- $Q_t(t)$ – transportation batch quantity at time .
- $B(t)$ – number of products with a pending order status at time .
- $LS(t)$ – lost sales quantity at time .
- – order batch lead time.
- – recovery batch lead time.
- – production batch lead time.
- – cost of launching the purchasing process.
- – cost of launching the recovery process.
- – unit cost of returns storage.
- – unit cost of new items storage.
- – unit pending order cost.
- – unit lack-of-inventory cost.
- – unit recovery process cost.
- – unit disposal process cost.
- – unit ordering process cost.
- – unit transportation cost.

2. INTRODUCTION

Reverse logistics understood as the process of managing reverse flow of materials, in-process inventory, finished goods and related information has become one of the logicians' key areas of interest. It enjoys ever-increasing interest of many industrial branches. Nowadays a growing number of companies realize the meaning of that field of logistics.

Inventory management is paid a great deal of attention to in works on the issue. A lot of mathematical models referring to that field have been designed so far.[2,7]

This article's purpose is to present models being modifications of a classical inventory control model in a continuous review system. The article focuses on the models presented in literature by particular authors.

3. AN ANALYSIS OF THE MODELS

Heyman in his work [3] analyzes a system where demand $D(t)$ for new products can be satisfied with returns recovery or new products purchase. Demand for new items and the number of returns $Z(t)$ are independent random variables described by the Poisson distribution with parameters λ_D and λ_z. There is one warehouse in which the returns are stocked in Heyman's system. Purchasing and recovery lead time is not taken into account. New items are not stored. If the number of products available in the warehouse is not sufficient to meet the current demand then the purchasing process is launched. The purchase batch quantity and the recovery batch quantity are not determined. The author doesn't take into consideration the cost of launching the recovery process and fulfilling the external orders. He allows that returns rejection is possible. Returns are being disposed of if on launching the recovery process the state of on-hand inventory $I_{poz}(t)$ equals value s_u. On-hand inventory is the currently available inventory increased by the orders placed and decreased by the pending orders. In Heyman's model:

$$I_{poz}(t) = I_z(t),$$

where:

$I_z(t)$ – returns inventory available at time .

The author takes into account the unit returns disposal cost k_u. He assumes, however, that there is always dependence:

$$K_{zam} - k_r - k_u > 0,$$

where:

– unit recovery process cost,

– unit ordering process cost.

Heyman takes into account a discount factor χ in his model. The joint cost function at time t will be as the following:

$$TC(t) = \begin{bmatrix} k_{zam} \cdot TQ_{zam}(t) + k_r \cdot TQ_{odz}(t) + \\ -k_u \cdot TQ_u(t) + k_{skz} \int_0^t I_z(x)\,dx \end{bmatrix},$$

where:

$TQ_{zam}(t)$ – the total number of products purchased from the outside at time t,

$TQ_{odz}(t)$ – the total number of products made subject to recovery processes at time t,

$TQ_u(t)$ – the total number of products made subject to disposal processes at time t,

The author minimizes the expected value of the system performance total cost. Heyman solves the problem using the theory of mass service. He notes that $S_N(t) = s_u - I_z(t)$ corresponds to queuing system realization $M/M/1/s_u$ with one service position and limited queue size equal to s_u. Requests received by the queuing system correspond to demand satisfied with recovery of products stocked in the returns warehouse. Accepting every request makes one bear the recovery cost k_r. Queuing system requests are rejected at intervals at which $S_N(t) = s_u$. It's like a situation when the returns warehouse is empty and demand is met through new items purchase. In a queuing system $I_z(t)$ corresponds to the number of spare places at time t. Queuing system idle periods when $S_N(t) = 0$ correspond to a situation in which the returns warehouse is replenished, there is no demand and all the incoming returns are disposed of. In order to specify the number of returns that are to be disposed of the author assumes that there is no system idle time, yet marginal tasks are being performed. The system functions as a queue with unlimited number of places until the next demand occurrence. Traffic intensity in Heyman's system is described in the following way:

$$\rho = \frac{\lambda_D}{\lambda_Z}.$$

The model worked out by Heymana was further developed by John A. Muckstadt and Michael H. Isaac. The authors analyze one- and two-echelon systems.

In one-echelon system, like in Heyman's system, demand for finished products and returned products are independent random variables described by the Poisson distribution with parameters λ_D and λ_Z. The authors assume that $\lambda_D > \lambda_Z$. That's why it's necessary to purchase new items. Unlike Heyman, the authors take into account the purchasing lead time. The items are supplied in L_{zam} time units. All the returned products need to be recovered. Recovery is performed according to the FIFO queuing system. Recovery lead time is an independent random variable. Recovery process products are brought to the finished products warehouse. The authors consider one warehouse for recovery process products and the products supplied within the purchase order framework. The authors allow that the lack of inventory is possible. Orders that were not fulfilled gain a pending status. On-hand inventory at time is defined as follows:

$$I_{poz}(t) = I_{nett}(t)+I_z(t)+Q_{zam}(t),$$

where:

$I_{nett}(t)$ – net inventory quantity at time t,

$I_z(t)$ – number of products found in the queuing system at time t, i.e. waiting for recovery,

$Q_{zam}(t)$ – order batch quantity being underway at time t.

$$I_{nett}(t)+I_{np}(t) - B(t)$$

where:

$I_{np}(t)$ – inventory available in the finished products warehouse at time t,

$B(t)$ – number of products that have a pending order status at time t.

The authors notice that as the time intervals between subsequent demand occurrences and returns introduction are described by the exponential distribution then it's possible to formulate a Markov chain for on-hand inventory.

As the lead time is invariable and all the orders placed at time $t - L_{zam}$ are available at time t, the net inventory quantity can be described in the following way:

$$I_{nett}(t) = \begin{bmatrix} I_{poz}(t-L_{zam}) - I_z(t-L_{zam}) + \\ +Q_{odz}(t-L_{zam},t) - D(t-L_{zam},t) \end{bmatrix},$$

where:

$Q_{odz}(t - L_{zam,t})$ – number of products leaving the recovery center at time interval $(t - L_{zam,}t]$,

$D(t - L_{zam,t})$ – quantity demanded at time interval $(t - L_{zam,}t]$.

The authors approximate the net inventory quantity by normal distribution. They assume that new items are ordered straight away at number $Q_{zam} \geq 1$ when on-hand inventory state in the finished products warehouse falls below value s_p+1. The purpose of their analysis is to determine optimal values s_p and Q_{zam} on the assumption that the recovery system is permanently at work.

The minimized objective function in a cost model is the following:

$$\begin{aligned} TC = {} & \frac{(\lambda_D - \lambda_Z)\cdot K_{zam}}{Q_{zam}} + \\ & +(k_B + k_{sknp})\left[\sigma\cdot\varphi\left(\frac{\mu}{\sigma}\right) - \mu\cdot\Phi\left(-\frac{\mu}{\sigma}\right)\right] + , \\ & +k_{sknp}\left(s_p + \frac{Q_{zam}}{2} + c\right) \to \min \end{aligned}$$

where:

$$\mu = s_p + \frac{Q_{zam}}{2} + c,$$

$$\sigma^2 = \frac{Q_{zam}^2}{12} + c_1,$$

$$c = \frac{\lambda_Z}{\lambda_D - \lambda_Z} + \frac{1}{2} - E(I_z(t)) - (\lambda_D - \lambda_Z)L_{zam},$$

$$\begin{aligned} c_1 = {} & \frac{\lambda_Z\cdot\lambda_D}{(\lambda_D - \lambda_Z)^2} - \frac{1}{12} + \\ & +Var(I_z(t)) + (\lambda_D + \lambda_Z)L_{zam} \end{aligned},$$

– unit pending order cost,

μ and δ^2 are the mean and the variance of the normal distribution which describes the net inventory quantity. Whereas $\varphi(\cdot)$ and $\Phi(\cdot)$ are the probability density function and the distribution function of a standard normal distribution respectively.

A central warehouse in which recovery processes are being carried out is at the upper echelon in a two-echelon system. New items purchased from the outside suppliers are delivered to that warehouse as well. The lower echelon consists of S retailers who have only warehouses. Demand $D^j(t)$ and returns $Z^j(t)$ directed to particular retailers are described by the Poisson distribution with parameters λ^j_D and λ^j_Z where $j=1,2,...,S$. The returns are immediately passed on to the central warehouse where they undergo recovery processes according to the FIFO queuing system. Returns to the central warehouse $Z(t)$ are described by the Poisson distribution with the following parameter:

$$\lambda_Z = \sum_{j=1}^{S} \lambda_Z^j .$$

Recovery process products don't need to be brought to the same retailer who passed them on to the central warehouse. The authors assume that the goods can't be passed on between particular retailers. Lead time L_c from the central warehouse to the warehouse at the lower echelon is invariable and it's the same for all the retailers.

The authors also assume that every -th retailer applies a continuous order policy ($s^j_p - 1$, s^j_p).

A retailer orders one item from the central warehouse at a time as soon as there is demand for it. Owing to that the demand in the central warehouse $D(t)$ is described by the Poisson distribution with the following parameter:

$$\lambda_D = \sum_{j=1}^{S} \lambda_D^j .$$

The authors assume different storage costs for goods in the central warehouse k_{sknp} and in the j-th retailer's warehouse k^j_{sknp} and different pending order costs k^j_B for individual retailers. The purpose of their analysis is to determine optimal values Q_{zam}, s_p and s^j_p. Muckstadt and Isaac formulate the following optimization problem:

$$\min_{Q_{zam}, s_p, s_p^j} \left\{ \begin{array}{l} \sum_{j=1}^{S} \left(k_{sknp}^j \cdot E\left(I_{np}^j(t) \right) + k_B^j \cdot E\left(B^j(t) \right) \right) + \\ + K_{zam} \cdot \dfrac{\lambda_D - \lambda_Z}{Q_{zam}} + k_{sknp} \cdot E\left(I_{np}(t) \right) \end{array} \right\},$$

where:

$Q_{zam} \geq 1$, $s_p \geq 1$, and $s^j_p = 0,1,\ldots,$

In their analysis the authors present an algorithm that can solve the existing problem. The algorithm compares the storage cost in the central warehouse with the storage cost and pending order cost at individual retailers'.[5]

The presented above multi-echelon model developed by John A. Muckstadt and Michael H. Isaac doesn't allow for disposal. Aybek Korugan and Surendra M. Gupta are the authors who eliminate that constraint. Demand and returns are still described by the Poisson distribution. The authors assume in their calculations that there is one retailer who the returns are handed over to according to parameter $\lambda^j_Z = \Sigma\ \lambda^j_Z$. Korugan and Gupta create a queuing system model in which the returns are collected and stored in the retailer's warehouse. At definite intervals of time the products are transported to the returns warehouse located near the workshop in which the recovery processes are carried out. Transportation time is the service time for the first queue position and it is described by the exponential distribution with a parameter λ_t. The recovery process efficiency is described by the exponential distribution with a parameter λ_{odz}. Recovery process products are stored in the finished products warehouse to which the demand $D(t)$ is directed to. The authors assume that $\lambda_D > \lambda_Z$. The difference between these parameters $Q_{prod} = \lambda_D - \lambda_Z$ describes the production process outcome. The sizes of the system warehouses are limited. They are respectively LM_1, LM_2 and LM_3 for succeeding warehouses. The authors don't take into consideration the pending orders. If the finished products warehouse is empty when there is demand for a product then the order is lost. The disposal process is launched when the retailer's returns warehouse is replenished. Korugan and Gupta create the following cost model:

$$TC = \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[\begin{array}{l} k_{skz}^s \cdot I_z^s(t) + k_{skz}^w \cdot I_z^w(t) + \\ + k_{sknp} \cdot I_{np}(t) + k_u \cdot Q_u(t) + \\ + k_b \cdot LS(t) + \\ + k_t \cdot Q_t(t) + \\ + k_r \cdot Q_{odz}(t) + k_p \cdot Q_{prod}(t) \end{array} \right] dt ,$$

where:

k^s_{skz} – cost of storage in the retailer's returns warehouse,

– cost of storage in the workshop's returns warehouse,

– cost of storage in the finished products warehouse,

– lost sales cost,

– transportation cost,

$I^s{}_z(t)$ – inventory quantity in the retailer's returns warehouse at time t,

$I^w{}_z(t)$ – inventory quantity in the workshop's returns warehouse at time t,

$Q_u(t)$ – disposal batch quantity being underway at time

$LS(t)$ – lost sales quantity at time t,

$Q_t(t)$ – transportation batch quantity at time t,

$Q_{odz}(t)$ – recovery batch quantity being underway at time t

$Q_{prod}(t)$ – production batch quantity being underway at time t

Korugan and Gupta analyze the presented above model using the expansion method. They check the impact of separate parameters on the cost quantity. They do that on the assumption that $k_t+k_r<k_p+k_u$.[4]

John A. Muckstadt and Michael H. Isaac's work is further developed by a group of authors among whom there are Ervin van der Laan, Rommert Dekker, Marc Salomon and Ad Ridder. The authors analyze a single-echelon system. Demand and returns are random quantities described by the Poisson distribution with corresponding parameters, like in the predecessors' analysis. The authors develop two approximation procedures and compare them with that of Muckstadt and Isaac. Van der Laan, Dekker, Salomon and Ridder presume in the first approximation that net inventory has the normal distribution when the products are supplied from the outside. As for the second approximation, it describes the difference between the demand and the recovery process outcome with the help of the Theory of Brownian motion.

The authors note that the increase of the returns number doesn't lead to average costs decrease. It's due to the growing holding cost of the returned products. That's why they suggest that some part of returns should be disposed of. The returns are disposed of at the workshop level at which the recovery processes are carried out. But the products located in the finished products warehouse are not disposed of. The decision about the returned product disposal is made up on the basis of the information about the number of products waiting for recovery. The authors assume that the workshop consists of C_q parallel positions with service time described by the exponential distribution. There is a reception room in the workshop. If there are s_q products in the reception room then every succeeding product is disposed of. [10]

In their subsequent research van der Laan and Salomon analyze the previously developed inventory management strategies in reverse logistics. They compare their own strategy with those of Heyman, Muckstadt and Isaac. They work out a policy based on four parameters , , , in order to achieve that. The disposal in their strategy is carried out in two cases: when on-hand inventory level reaches value and when the number of products waiting for recovery in the workshop equals . The policy (s_p,Q_{zam},s_u,s_q) and its versions (s_p,Q_{zam},s_u) and (s_p,Q_{zam},s_q) are subject to numerical comparison. The authors prove that the strategy (s_p,Q_{zam},s_u,s_q) allows to achieve the lowest cost. Highly-complicated calculations used to find an optimal solution are the policy's drawback. The authors claim that the mentioned above versions (s_p,Q_{zam},s_u) may be of better practical use. They show that the first version allowed them to achieve lower costs in most comparisons that had been made. [9]

Ervin van der Laan deals with the comparative analysis of pull and push systems in reverse logistics in his works as well. He presents, along with M. Salomon, a simplified version of a system used in practice by photocopier manufacturers.

The system contains a returns warehouse and a finished products warehouse where new items and recovery process products are stored. The analyzed product consists of one module. All the returned products are recoverable. Some part of products placed in the returns warehouse is subject to disposal. Time intervals between subsequent demand occurrences and subsequent returns occurrences are described with the help of the Cox distribution. The authors assume that returns and demand are correlated which means there is some probability that product returns will result in demand occurrence. The authors allow that the lack of inventory is possible which makes pending orders appear. They assume that production and recovery lead time is invariable.

In a push system the authors use policy $(s_p,Q_{prod},Q_{odz},s_u)$ in which the recovery process is launched when the inventory level in the returns warehouse reaches value Q_{odz}. The process of items manufacture is launched when on-hand inventory level in the finished products warehouse falls to items. The disposal is launched if on-hand inventory level is higher than or equals items.

In a pull system the authors suggest policy (s_p,Q_{prod},S_r,s_u) in which the recovery process is launched when the on-hand inventory level in the finished products warehouse is less than or equals s_r and the number of products in the returns warehouse is sufficient to increase the on-hand inventory level to value S_r items. Q_{prod} items manufacture is launched when on-hand inventory level falls to s_p items. The disposal is launched if inventory level in the returns warehouse reaches value s_u.

Having made the comparative analysis of the presented systems, the authors state that the pull system is more cost-effective only on condition that the cost of holding inventory in the returns warehouse is significantly lower than that of holding it in the finished products warehouse.[8]

Moritz Fleischmann, Reolof Kuik and Rommert Dekker are the following authors who develop inventory management theory in reverse logistics. They design a simple inventory management model (s_p,Q_{zam}) in reverse logistics. The authors go back in their research to John A. Muckstadt and Michael H. Isaac's model. They analyze a simplified case in which returned products are subject to immediate use and are stored in the same warehouse as new items. New items are obtained through a purchase. The lead time is invariable. Demand and returns are independent and they are described by the Poisson distribution. The authors note that the number of returns could be modeled in the previous demand occurrence function. Nevertheless, they admit that estimating that kind of dependence in practice is extremely difficult. Non-fulfilled orders acquire the pending status. The authors take into consideration the cost of launching the purchase order, unit cost of products storage and of pending orders fulfillment. The purpose of the research is to determine an optimal policy that would minimize the average cost function during a specified period. Fleischmann, Kuik and Dekker's model is a simple development of a classical inventory management model.[1]

In their succeeding work dealing with inventory management in reverse logistics Huiqing Ouyang and Xiangyang Zhu note that most previously developed models are based on the assumption that the number of returns doesn't exceed the demand. They claim that the existing models don't describe the final stage of the product life cycle when the number of returns can significantly exceed the demand for them. Huiqing Ouyang and Xiangyang Zhu introduce the inventory management policy (s_p,Q_{zam},s_u) which allows that it's possible. One of Ervin van der Laan's works presents the same policy but it doesn't consider the returns storage cost. The model singles out two warehouses: a returns warehouse and a finished products warehouse. The finished products warehouse is replenished owing to returns value recovery or raw materials purchase and new items manufacture. The recovery has higher priority like in most preceding analyses. Demand and returns remain independent and they are described by the Poisson distribution. Product recovery is modeled according to the FIFO queuing system. Recovery time is described by the exponential distribution. On-hand inventory at time t is calculated in the same way as in John A. Muckstadt and Michael H. Isaac's model. Huiqing Ouyang and Xiangyang Zhu allow that pending orders may occur but only in a case when the finished products warehouse is to be replenished with manufacture. The authors also assume that not more than one purchase order can be open at a time.[6]

4. SUMMARY

The article deals with stochastic inventory management models in reverse logistics systems based on a continuous review. The analysis presents particular authors' contribution to the development of reverse logistics theory. Their successors eliminate individual constraints creating more and more complicated mathematical models.

The presented models are based on the assumption that demand for finished products can be satisfied with returns recovery, new items production or procurement. All the presented models assume that demand and returns have random character. In most analyses demand and returns are totally independent and described by the Poisson distribution with corresponding parameters. All the presented models deal with a single-item product. Some models focus on multi-echelon systems. Recovery process is modeled with the help of the mass service theory. The time of launching recovery, manufacture or purchasing processes is dependent on the corresponding information levels. On-hand inventory quantity is monitored in warehouses.

BIBLIOGRAPHY

[1] Fleischmann, M., Dekker, R., Kuik, R. *Controlling inventories with stochastic item returns: A basic model.* European Journal of Operational Research, 138, s. 63–75 (2002)

[2] Fleischmann, M., Bloemhof–Ruwaard, J. M., Dekker, R., van der Laan, E., van Nunen, J. A. E. E., Van Wassenhove, L. N. *Quantitative models for reverse logistics: A review.* European Journal of Operational Research, 103 (1997), 1–17.

[3] Heyman, D.P, *Optimal Disposal Policies for a Single–Item Inventory System with Returns*, Naval Research Logistics Quarterly, 24, s. 385–405 (1977)

[4] Korugan, A., Gupta, S.M., *A Multi–Echelon Inventory System with Returns.* Computers & Industrial Engineering, 53, s. 145–148 (1998)

[5] Muckstadt, J.A., Isaac, M.H., *An analysis of single item inventory systems with returns.* Naval Research Logistics Quarterly, 28, s. 237–254 (1981)

[6] Ouyang, H., Zhu, X., *An Inventory Control System for Remanufacture with Disposal.* RAM 2006, s. 1–5 (2006)

[7] Thierry, M.C., Salomon, M., Van Nunen, J., Van Wassenhove, L., 1995. *Strategic issues in product recovery management.* California Management Review 37, 114–135.

[8] Van der Laan, E.A., Salomon, M., *Production planning and inventory control with remanufacturing and disposal.* International Journal of Operational Research, 102, s. 264–278 (1997)

[9] Van der Laan, E.A., Dekker, R., Salomon, M., *Product remanufacturing and disposal: A numerical comparison of alternative control strategies.* International Journal of Production Economics, 45, s. 489–498 (1996)

[10] Van der Laan, E.A., Dekker, R., Salomon, M., Ridder, A., *An (S,Q) inventory model with remanufacturing and disposal.* International Journal of Production Economics, 46–47, s. 339–350 (1996)

An analysis of continuous review inventory control models in reverse logistics

Abstract

"Reverse logistics encompasses the logistics activities all the way from used products no longer required by the user to products again usable in a market".[2]

In reverse logistics systems demand can be satisfied with production or procurement and any kind of reuse option. The used products are brought back, stored and reused in due time to satisfy the demand. Same part of this flow can be also disposed of. Inventory management has a significant meaning in reverse logistics.

The goal of his paper is to investigate continuous review models in reverse logistics. First model in category was created by Heyman in 1977. Along the same line of research, new models were created by Muckstadt, Korugan, Fleischmann and van der Laan.