An Analysis of Stochastic Inventory Control Models in Reverse Logistics Systems Based on a Periodic Review

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In reverse logistics systems demand can be partially satisfied with new items manufacture or procurement and returned products value recovery. The products are brought back to the places where they are stored in most models presented in literature on reverse logistics. Value recovery processes are carried out in due time in order to meet the existing demand. Some part of returns can be disposed of.
Inventory management has significant meaning in reverse logistics. This article's purpose is to present models being modifications of a classical inventory control model in a periodic review system. The first model of that kind was developed by Simpson in 1978. Guided by similar assumptions, Inderfurth, Kiesmuller, Minner and Cohen, among others, designed periodic review models as well.

1. BASIC NOTATIONS

- D_t – quantity demanded for finished products at time t.
- Z_t – quantity of returns at a period t.
- T – planning horizon scope.
- $I_{poz,t}$ – on-hand inventory quantity in the finished products warehouse.
- $I_{z,t}$ – inventory state in the returns warehouse at a period t.
- $I_{np,t}$ – inventory state in the finished products warehouse at a period t.
- Q_{zam} – optimal order batch quantity.
- $Q_{zam,t}$ – order batch quantity at a period t.
- Q_{prod} – optimal production batch quantity.
- $Q_{prod,t}$ – production batch quantity at a period t.
- Q_{odz} – optimal recovery batch quantity.
- $Q_{odz,t}$ – recovery batch quantity at a period t.
- Q_u – optimal disposal batch quantity being underway.
- $Q_{u,t}$ – disposal batch quantity at a period t.
- B_t – number of products with a pending order status at a period t
- L_{zam} – order batch lead time.
- L_{odz} – recovery batch lead time.
- L_{prod} – production batch lead time.
- k_{skz} – unit cost of returns storage.
- k_{sknp} – unit cost of new items storage.
- k_B – unit pending order cost.
- k_b – unit lack-of-inventory cost.
- k_r – unit recovery process cost.
- k_u – unit disposal process cost.
- k_{zam} – unit ordering process cost.

2. INTRODUCTION

Reverse logistics understood as the process of managing reverse flow of materials, in–process inventory, finished goods and related information has become one of the logicians' key areas of interest. It enjoys ever–increasing interest of many industrial branches. Nowadays a growing number of companies realize the meaning of that field of logistics.

Inventory management is paid a great deal of attention to in literature on the issue. A lot of mathematical models referring to that field have been designed so far.[3,11]

This article's purpose is to present models being modifications of a classical inventory control model in a periodic review system. The article describes the models presented in literature by particular authors.

3. AN ANALYSIS OF THE MODELS

The first model dealing with a periodic review inventory management system was worked out by V. P. Simpson. In Simpson's model demand for new products D_t and the number of returns Z_t at a period t, where $t=1,2,3,\ldots,T,$ are independent random variables. The known probability density function of two variables $\Omega_t(D_t,Z_t)$ is the only thing that connects those variables. There are two warehouses in Simpson's model: a returns warehouse and a finished products warehouse. Inventory level $I_{np,t}$ in the finished products warehouse and inventory level $I_{z,t}$ in the returns warehouse are checked over at the beginning of each period. The warehouse inventory level can change as a result of new items purchase in quantity $Q_{zam,t}$, returns recovery in quantity $Q_{odz,t}$ and returned products disposal in quantity $Q_{u,t}$. Purchasing and recovery lead time is not taken into account. One item purchasing and recovery costs are fixed: k_{zam} and k_r respectively. The author doesn't take into consideration the cost connected with disposal. Unfulfilled demand quantity is monitored in Simpson's model. Storage cost for goods in a finished products warehouse is calculated for each period. The function of the unfulfilled demand expected cost and the expected storage cost in a finished products warehouse for period t is the following:

$$TC_{1,t}\left(I_{np}\right)=k_{snp}\int_0^{\infty}\int_0^{I}\left(I_{np}-x\right)\Omega_t\left(x,y\right)dxdy+ +k_b\int_0^{\infty}\int_I^{\infty}\left(x-I_{np}\right)\Omega_t\left(x,y\right)dxdy,$$

where:

k_{snp} – one item storage cost in the finished products warehouse,

k_b – lack–of–inventory cost,

I_{np} – initial inventory state in the finished products warehouse,

I_z – initial inventory state in the returns warehouse.

One returned item storage cost in the returns warehouse is k_{skz}. The total returns storage cost $TC_{2,t}(I_z)$ is calculated analogically to the cost of storage and delay in the finished products warehouse. Simpson uses dynamic programming in order to determine an optimal cost quantity. He formulates the following optimization problem:

$$TC\left(I_{np},I_z\right)=\min\left\{\begin{array}{l} k_rQ_{odz,t}+k_{zam}+ \\ +TC_{2,t}\left(I_z-Q_{odz,t}-Q_{u,t}\right)+ \\ +TC_{1,t}\left(I_{np}+Q_{odz,t}+Q_{zam,t}\right)+ \\ \mu\int_0^{\infty}\int_0^{\infty}A_{t-1}\begin{pmatrix} I_{np}+Q_{odz,t}+ \\ +Q_{zam,t}-x, \\ I_z-Q_{odz,t}- \\ +Q_{u,t}+y \end{pmatrix}\Omega_t dydx \end{array}\right\}$$

Recursive dependence between inventory levels in the finished products warehouse and in the returns warehouse at particular periods is the following:

$$I_{np,t-1}=I_{np,t}+Q_{odz,t}+Q_{zam,t}-D_t,$$

$$I_{z,t-1}=I_{z,t}-Q_{odz,t}-Q_{u,t}+Z_t.$$

Simpson works out a policy ($S_{zam,t}$, $S_{odz,t}$, $S_{utyl,t}$) according to which, if the inventory level in the finished products warehouse is lower than $S_{odz,t}$ then the inventory is replenished owing to returns recovery. If, on having decided to launch the recovery process, the inventory level in the returns warehouse and in the finished products warehouse is lower than $S_{zam,t}$ then the inventory is replenished owing to new products purchase. If after those decisions the joint inventory in the returns warehouse and in the finished products warehouse is larger than $S_{odz,t}+S_{utyl,t}$ then the inventory disposal process is launched to reach the level $S_{odz,t}+S_{utyl,t}$

The mentioned above policy for $S_{zam,t},S_{odz,t},S_{utyl,t}\geq 0$ can be presented in the following way:

$$Q_{odz,t}\left(I_{np},I_z\right)=\begin{cases} 0 & dla\ I_{np}\geq S_{odz,t} \\ S_{odz,t}-I_{np} & dla\ I_{np}<S_{odz,t} \\ & oraz \\ & I_{np}+I_z\geq S_{odz,t} \\ I_z & dla\ I_{np}+I_z<S_{odz,t} \end{cases}$$

$$Q_{zam,t}\left(I_{np},I_z\right)=\begin{cases} 0 & dla\ I_{np}+I_z\geq S_{zam,t} \\ S_{zam,t}-I_{np}-I_z & dla\ I_{np}+I_z<S_{zam,t} \end{cases}$$

$$Q_{u,t}\left(I_{np},I_z\right)=\begin{cases} 0 \quad dla \quad I_{np}+I_z\leq S_{odz,t}+S_{utyl,t} \\ \min\left(I_z,I_z+I_{np}-S_{odz,t}-S_{utyl,t}\right)\ dla \\ I_{np}+I_z>S_{odz,t}+S_{utyl,t} \end{cases}$$

Simpson uses the Kuhn-Tucker conditions in order to determine the minimum of joint cost function.[9]

Karl Inderfurth develops a model based on similar assumptions. Unlike his predecessor, he takes into account recovery process lead time L_{odz} and new products lead time L_{zam} expressed in the number of periods. Unfulfilled demand takes the form of pending orders. Inderfurth analyzes two cases in his works. He doesn't allow for returns storage in the first one. He eliminates that constraint in the second one. Inderfurth aims to minimize joint costs for all the periods at a particular planning horizon T.

$$TC = E\left\{\sum_{t=1}^{T}\begin{bmatrix} TC_{zou}\left(Q_{zam,t}, Q_{odz,t}, Q_{u,t}\right) + \\ + K_{skz}\left(I_{z,t}\right) + K_{sknp}\left(I_{np,t}\right) \end{bmatrix}\right\} \rightarrow \min ,$$

,

where:

$TC_{zou}(Q_{zam,t}, Q_{odz,t}, Q_{u,t})$ – purchase, recovery and disposal joint costs at a period t,

$K_{sknp}(I_{np,t})$ – joint costs of finished products storage at a period t,

$K_{skz}(I_{z,t})$ – joint costs of returns storage at a period t,

$I_{np,t}$ – available inventory quantity in the finished products warehouse at the end of period t,

$I_{z,t}$ – available inventory quantity in the returns warehouse at the end of period t,

$$I_{np,t} = I_{np,t-1} + Q_{zam,t-L_{zam}} + Q_{odz,t-L_{odz}} - D_t ,$$

$$I_{z,t} = I_{z,t-1} + Z_t - Q_{odz,t} - Q_{u,t} .$$

Inderfurth notes that the difference between L_{odz} and L_{zam} is a factor that has significant impact on the model's level of complication. Inderfurth works out an optimal inventory carrying policy in a case when that difference is smaller or equal during one period.

In the simplest case when the returns are not stored and $L_{odz} = L_{zam} = L$. Inderfurth uses policy (S_t, s_{ut}) where $-\infty \le S_t \le s_{u,t} \le +\infty$. According to that policy if on-hand inventory is lower than S_t then the recovery process of all the warehouse returns is launched. If on recovery process completion the on-hand inventory level $I_{poz,t}$ is still lower than S_t then new products are purchased. A disposal process is launched if the on-hand inventory level is higher than $s_{u,t}$. Excess inventory is disposed of. The remaining returns are subject to recovery processes. The on-hand inventory is determined on the basis of the current inventory state, returns from the previous period, purchase and recovery orders being underway, as well as inventory reservations within the pending order framework.[4]

G.P. Kiesmuller and S. Minner are the following authors dealing with inventory control in reverse logistics systems. In their works [6,7] the authors present a model in which demand for finished products D_t at particular periods is an independent random variable with the same probability distribution. The distribution function F_D and the expected value λ are known. Unfulfilled demand takes the form of pending orders. Returns Z_t are described analogically to demand. The distribution function F_Z and the expected value λ_Z are known. The difference between demand and returns is described by the distribution function F_{D-Z}. The authors assume that returns don't depend on demand. Kiesmuller and Minner don't consider disposal. They assume that all the returns are recoverable. The authors model a production system in which the finished products warehouse is replenished with production and recovery. They take into account production and recovery lead time. They assume that a review takes place at the beginning of each period. The number of products that are to be manufactured and recovered is dependent on policy (S_{prod}, S_{odz}). Decisions about production and recovery quantity are made simultaneously but recovery has a higher priority. The on-hand inventory quantity $I_{pozp,t}$ is used to define the production quantity. Recovery quantity is defined with the help of on-hand inventory $I_{poz,t}$

$$Q_{prod,t} = \left(S_{prod} - I_{pozp,t}\right)^+ ,$$

$$Q_{odz,t} = \min\left\{I_{z,t}, \left(S_{odz} - I_{poz,t}\right)^+\right\} ,$$

where:

$I_{z,t}$ – available returns inventory at a period t.

In a case when $L_{odz} = L_{prod} = L$ the authors define $I_{poz,t}$ as a sum of available finished products inventory and production and recovery orders, which are being launched, decreased by pending orders. As recovery has a higher priority, one should additionally consider the available returns inventory while defining $I_{pozp,t}$.

$$I_{pozp,t} = I_{z,t} + I_{np,t} + \sum_{i=1}^{\bar{}} \left(Q_{prod,t-i} + Q_{odz,t-i} \right).$$

$$I_{poz,t} = I_{np,t} + \sum \left(Q_{prod,t-i} + Q_{odz,t-i} \right).$$

In a case when $L_{odz} > L_{prod}$:

$$I_{pozp,t} = I_{np,t} + \sum_{i=1}^{L_{prod}} Q_{prod,t-i} + \sum_{i=0}^{L_{odz}} Q_{odz,t-(L_{odz}-L_{prod}+i)},$$

$$I_{poz,t} = I_{np,t} + \sum_{i=0}^{L_{prod}} Q_{prod,t-i} \sum_{i=1}^{L_{odz}} Q_{odz,t-i}.$$

In a case when $L_{odz} < L_{prod}$:

$$I_{pozp,t} = I_{np,t} + I_{z,t} + \sum_{i=1}^{L_{prod}} Q_{prod,t-i} + \sum_{i=1}^{L_{odz}} Q_{odz,t-i},$$

$$I_{poz,t} = I_{np,t} + \sum_{i=0}^{L_{prod}} Q_{prod,t-(L_{prod}-L_{odz}+i)} \sum_{i=1}^{L_{odz}} Q_{odz,t-i}.$$

Kiesmuller and Minner develop the following cost model:

$$TC = k_{sknp} E\left[I_{np}^{+}\right] + k_{skz} E\left[I_{z}\right] + k_{B} E\left[I_{np}^{-}\right] \rightarrow \min.$$

Mahadevan, Pyke and Fleischmann model a production system in which returns and demand are described by the Poisson distribution with parameters λ_D and λ_Z. The system consists of a returns warehouse and a finished products warehouse. Storage costs in the mentioned above warehouses are different. The authors take into account new products lead time and recovery process duration. They are constant. Unfulfilled demand takes the form of a pending order.

In the system described above a review is performed every R periods. All the returns that are in the returns warehouse at a given point in time undergo the recovery process. Recovery batch quantity $Q_{odz,t}$ is thus a random quantity. New items production is launched if on-hand inventory level $I_{poz,t}$ at the moment of review is lower than the target inventory level S_{prod}. Production batch quantity $Q_{prod,t}$ equals the difference between those quantities. The authors minimize the joint cost function by choosing the appropriate value of the target inventory level S_{prod}. The change of a variable value R helps to regulate the production batch quantity and the recovery batch quantity. The authors assume that R is given. They use heuristics in their calculations. They create a simulation model PROMODEL. The joint cost function takes the following form:

$$TC = k_{sknp} \overline{I}_{np} + k_{skz} \overline{I}_{z} + k_{B} \frac{\overline{B}}{R},$$

where:

$\overline{I}_{np}$ – average inventory level in the finished products warehouse,

$\overline{I}_{z}$ – average inventory level in the returns warehouse,

$\overline{B}$ – average number of pending orders.[8]

M. A. Cohen, S. Nahmias and W. P. Pierskalla focus on a periodic review inventory control system in their work as well. Each damaged product is substituted with a new one in the system. Demand is thus equal to the number of returns. Damaged products are returned to a recovery center from which they are delivered to the finished products warehouse after time L_{odz} expressed in the number of periods. Some part ($1 - \delta$) of returned products is not recoverable and leaves the system. Products shortage is replenished with new items purchase. New products lead time is not considered. Demand at particular periods is an independent random variable but with the same probability distribution. ($1 - \delta_z$) of inventory is subject to spoilage at each period and is no longer stored. The authors don't take into account pending orders, excess demand is equated with lost sales. The orders being underway are not allowed in the on-hand inventory because new items are supplied when an order is placed. There are $I_{poz,t}$ items in the warehouse at the beginning of each period in the event that recovery process products have been delivered. $S_{zam,t}$ products are to be found in the warehouse when the order for new items is fulfilled. Order batch quantity $Q_{zam,t}$ equals $S_{zam,t} - I_{poz,t}$. The authors take into account one item order cost. Fixed cost of the order service isn't considered. The function of joint storage cost and lost sales cost for one period is the following:

$$TC_1 = E\left\{ \begin{array}{l} k_{sknp}\left[\left(S_{zam} - D\right)^{+}\right] + \\ +k_b\left[\left(D - S_{zam}\right)^{+}\right] + \\ +k_{zep}\left(1-\delta_z\right)\left(S_{zam} - D\right)^{+} \end{array} \right\} =,$$

$$\int_0^{I_{zam}} \left(S_{zam} - t\right) f(t)\,dt + $$

$$+\int_{I_{zam}}^{\infty} k_b \left(t - S_{zam}\right) f(t)\,dt +$$

$$+k_{zep}\left(1-\delta_z\right) \int_0^{I_{zam}} \left(S_{zam} - t\right) f(t)\,dt$$

where: k_{zep} – unit product spoilage cost.

In the total cost function TC the authors take into account order service costs as well. The work aims to determine an optimal value S_{zam} for all the planning horizon for which the total cost function TC reaches its minimum. The authors develop an order fulfillment optimal policy for a case when $L_{odz} = 1$. They specify an approximate solution for $L_{odz} > 1$.[2]

Peter Kelle and Edward A. Silver create a model similar to that of Cohen. The authors develop an optimal policy for new products purchase using the example of reusable packaging. Kelle and Silver analyze net demand $D_{nett,t}$ which at a period t is equal to the difference of actual demand D_t and the number of returned bags Z_t suitable for reuse. Demand and returns are independent random quantities. All the returns undergo recovery processes. The authors don't consider the disposal. Unfulfilled demand takes the form of pending orders and is satisfied during the further periods. The authors don't take into account the cost related with pending orders service. They substitute it with a consumer service demanded level which equals $(1-\varepsilon_t)$ for a period t. The authors try to minimize the joint cost of new bags purchase and returns storage while considering the demanded level of service. The planning horizon includes T periods. The purpose of the analysis is to specify an optimal order quantity $Q_{zam,t}$ where $t = i+1, i+2, ..., i+T$ and i defines the current period. The joint cost function is the following:

$$TC = \sum_{t=i+1}^{i+T} \begin{bmatrix} k_{zam,t} \cdot Q_{zam,t} + K_{zam,t} \cdot \sigma\left(Q_{zam,t}\right) + \\ + k_{sknp} \cdot E\left(I_{np}\right) \end{bmatrix} \rightarrow \min ,$$

where:

$\sigma(Q_{zam,t})=0$ if $Q_{zam,t}=0$ or $\sigma(Q_{zam,t})=1$ if $Q_{zam,t}>0$,

$I_{np,t} = max\{I_{nett},0\}$ – inventory available in the warehouse at the end of period t,

$I_{nett,t}$ – net inventory at the end of a period resulting from the difference between the inventory available in the warehouse and pending orders,

$$I_{nett,t} = I_{nett,t-1} + Q_{zam,t} - D_{nett,t} ,$$

$$Q_{zam,t} \geq 0 .$$

The authors assume that the probability of meeting the demand with what is stocked in the warehouse equals at least $(1-\varepsilon_t)$. It's shown in the following formula:

$$P\left(I_{nett,t} \geq 0\right) \geq 1 - \varepsilon_t .$$

Kelle and Silver state that net inventory is a sufficient approximation of the actual inventory quantity for the service level used in practice, which is from 0,9 to 0,95. The authors reduce the presented above stochastic model to a deterministic model. Only a deterministic model is analytically solved. The authors note that it's a classical problem of determining production batch quantities in a deterministic model with variable demand for definite periods.[5]

D. J. Buchanan and P. L. Abad describe reusable packaging inventory management system as well. The authors create a model similar to that of Kelle and Silver. Buchanan and Abad analyze single- and multi-period model. As for multi-period model, the authors assume that the number of returns Z_t at a period t is a fraction $a_{z,t}$ of all the products $A_{z,t}$ found on the market at the beginning of a period t. $a_{z,t}$ is a random variable with a probability density function $F_{z,t}(a_{z,t})$. At each period some part of products found on the market $(1-\delta)$ isn't suitable for reuse. The authors take into consideration the cost k_{khp} connected with products that were not sold at the end of the analyzed planning horizon. Demand for new package D_t is a random variable with a density function $f_{D,t}(D_t)$ and a distribution function $F_{D,t}(D_t)$. The authors aim to minimize the joint cost in the analyzed planning horizon. They specify an optimal value Q_{zam}. They use dynamic programming in order to solve the model. They assume that the time of product presence on the market is described by the exponential distribution [1].

The models presented above are based on the assumptions of a classical periodic review model. Subsequent authors R. H. Teunter and D. Vlachos developed a model which was the system modification. The authors developed a model similar to continuous review model that had been earlier introduced by Ervin Van der Laan and Marc Salomon. The authors analyze the system in a limited planning horizon consisting of T time units. They consider equal recovery and production lead time L which is a multiplicity of the accepted time units.

$$L_{odz} = L_{prod} = L$$

Both recovery order and production order are launched at the beginning of the period. The authors allow that returns may be disposed of. Disposal is carried out at the beginning of the period as well. Demand at each time unit is an independent random variable. The authors model the demand with the help of the Poisson distribution and the normal distribution. They allow that pending orders may occur. Returns are described in an analogous way. The authors assume that the returns warehouse and the finished products warehouse are empty and there are no orders underway at the first period. Only production can be launched at the first period. The authors consider fixed and variable costs of production and recovery processes, as well as disposal process variable costs. The authors consider equal carrying cost of goods stored in the returns warehouse and in the finished products warehouse. They also take into account a discount factor χ for costs in the analyzed planning horizon. The authors assume that recovery batch quantity Q_{odz} and production batch quantity Q_{prod} are invariable. The inventory control policy that have been used is based on the same reorder point for recovery and production. A new batch is manufactured or recovered if the on-hand inventory level at the beginning of the period is lower or equals s items, according to that policy. Recovery process is launched if there are at least Q_{odz} items in the returns warehouse. Otherwise, it is a sign for starting the production process. The disposal process is launched if there are at least s_u returned items in the returns warehouse at the beginning of the period after the possible decision to start the recovery. Excess returns are disposed of. Teunter and Vlachos analyze the model using a computer simulation.[10]

4. SUMMARY

The article deals with stochastic inventory management models in reverse systems based on a periodic review. The analysis presents particular authors' contribution to the development of reverse logistics theory. Their successors eliminate individual constraints creating more and more complicated mathematical models.

The presented models are based on the assumption that demand for finished products can be satisfied with returns recovery, new items production or procurement. All the presented models assume that demand and returns have random character. In most analyses demand and returns are totally independent and described by the Poisson distribution with corresponding parameters. All the presented models deal with a single-item product. In most works recovery, production, order or disposal batch quantities are determined on the basis of the current on-hand inventory state.

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